



psychology. In our studies, we employ non-stereotypic number line tasks as key contexts for assessment and instruction. For example, in *interview studies*, we have investigated students' reasoning about core mathematical ideas related to the representation of integers and fractions on the number line. *Tutorial studies* have revealed students' conceptual and representational resources and identified the promise of the role of mathematical principles and definition in supporting students' understanding of numerical units. *Classroom studies* have engaged us in a process of iterative refinement of lessons with the collaboration of our expert teachers.

Several strands run through all of the work represented in Figure 1.

*Non-routine problems.* In all of our studies and lessons, non-stereotypic number line tasks serve as key contexts for the construction and application of mathematical principles and definitions. Non-routine problems elicit student thinking rather than their memorized solutions to routine tasks. For example, students are asked to identify points on lines that are not partitioned equally, or to locate additional numbers on a line that is labeled with only two numbers (that differ by an integer value greater than 1), and to explain how their number line principles help them solve each task.

*Principles and definitions.* The principles and definitions approach emerged as a productive instructional strategy in our tutorial studies. In the lessons, teachers engage students with the construction and flexible use of core principles of number lines as these apply to integers and fractions. Examples of core principles and definition include order, unit interval (and the related ideas of subunit and multiunit), 0 as origin, symmetry, absolute value, denominator, numerator, and equivalent fraction. Principles and definitions support the treatment of mathematics as rooted in deductive reasoning and support students as they analyze their reasoning in whole class and partner discussions.

*Units and unitizing activities.* The flexible construction and use of units, multiunits, and subunits are core to an understanding of integers and fractions. In the tutorial studies, we introduced Cuisenaire™ rods as a key resource for supporting students' understandings of linear unit. Over a sequence of tutoring tasks, the function of Cuisenaire™ rods gradually shifted from physical materials to be recorded on an open number line (for example, show the length of 3 reds from 0 on the line) to rods as tools to measure lengths between intervals on the line. In this process, Cuisenaire™ rods become key resources in unitizing activities that support students' elaboration of linear units, multi-units (multiples of a linear unit), and subunits (fractional parts of a linear unit).

These strands are represented in the posters:

- Interview study: Meghan Shaughnessy will discuss findings from an interview study of students' understandings of linear unit. Our research strategy highlights the productive role of non-routine number lines in assessment.
- Tutorial study: I will present findings from a tutorial study on integers. This study illustrates both the shifting function of Cuisenaire rods to support unitizing activities, and the roles of principles & definitions in resolving discrepant ideas about number on the number line.

- Classroom studies: Ronli Diakow and Meryl Gearhart will share findings from studies in the classrooms of our two collaborating teachers. The poster illustrates the design of one lesson and typical patterns of student reasoning, and reports findings on student learning from gains on unit pre-post assessments.
- Lesson materials: Yasmin Sitabkhan will share examples of our lesson materials at a resource table. These materials illustrate the ways that we integrate non-routine problems as contexts for the construction and application of number line principles & definitions.