# Learning Mathematics through Representations (LMR): Interview Studies

## Overview of Studies

- The **Number Line** is a geometrical interpretation of number constituted by a straight line partitioned into linear units of measure.
- When equidistant tick marks are labeled on routine number lines (a), ideas about linear units, numerical units, and their coordination are not made explicit. Non-routine number lines (b) are promising representations for investigating student understanding.

(a) Routine Number Line (b) Non-routine Number Line

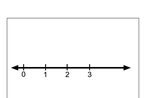
- **The Studies:** We conducted two coordinated studies to reveal patterns in 5th grade students' understandings.

## Study 1

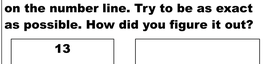
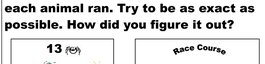
- Do fifth grade students coordinate linear and numerical units when locating integers on the number line?
- Are students more likely to coordinate units when lines represent real-world contexts, such as a race course?

## Method

- **Participants:**
  - 48 fifth grade students from 4 elementary schools in Northern California.
  - Students were matched on level of math achievement within classroom and then randomly assigned to the number line or the race course condition.
- **Interview:**

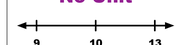
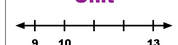
| Number Line | Race Course |
|---|---|
| This number line has 0, 1, 2, and 3 marked. Other numbers can go on the number line too.  | The turtle, the snail and the spider like to race and they measure the distances that they have run in units of wugs. This is one wug [distance from 0 to 1]. |

  - Students were introduced to the tasks as follows:
  - Three tasks (see instructions below):
    - Partial Consecutive Numbers: 9, 10, 13
    - Partial Consecutive Numbers: 9, 12, 13
    - All Non-Consecutive Numbers: 7, 11, 14

| Number Line | Race Course |
|---|---|
| Your job is to put these three numbers on the number line. Try to be as exact as possible. How did you figure it out?  | Your job is to show the distance that each animal ran. Try to be as exact as possible. How did you figure it out?  |

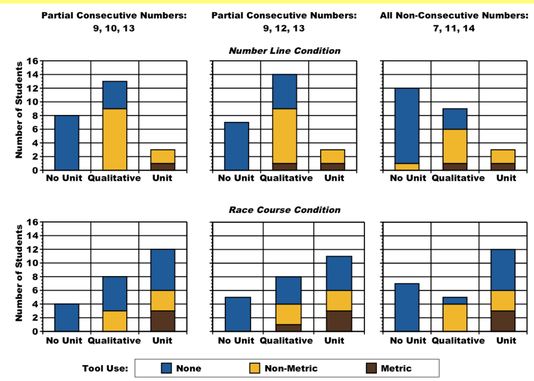
## Coding Scheme

- **Strategy:** Coordination of numerical & linear units.
  - **No Unit:** No evidence that the student constructed this coordination; students commonly placed the values in order and approximately equidistant.
  - **Qualitative:** Evidence that student conceptualized the number series in terms of relative magnitude, but the relative distances that they constructed were markedly inaccurate, and the student's explanation referred only to relative distances.
  - **Unit:** Numbers were placed so that the distances between points approximated the distances between target numbers.

| No Unit | Qualitative | Unit |
|---|---|---|
|  | |  |

- **Tool:** Whether students used improvised 'tools' to mediate their placements:
  - **None:** No evidence of tool use
  - **Non-metric:** Gestural activity, such as pulsing
  - **Metric:** Rigid object used as a measure (e.g., pen cap)

## Results



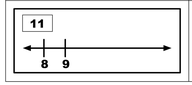
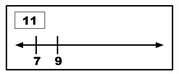
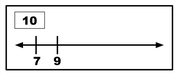

- **Strategies:** Main effect of task context ($p$ =.006): Students used Unit strategies more frequently in the Race Course Condition than the Number Line Condition.
- **Tool Use:** Students' use of tools varied independently of their use of a linear unit strategy.

## Study 2

- How do fifth grade students make use of a linear unit on the number line to place a third number?

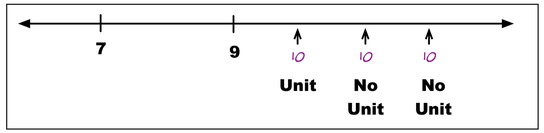
## Method

- **Participants:** 24 fifth grade students from 4 elementary schools in Northern California.
- **Procedure:** Students were asked to locate a number on a number line on which two numbers were already specified. Two forms each of three types of tasks.

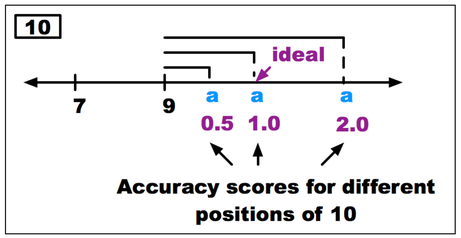
| Task Type | | |
|---|---|---|
| (i) Unit Interval: Multiunit Target | (ii) Multiunit Interval: Multiunit Target | (iii) Multiunit Interval: Fractional Target |
|  |  |  |

## Coding Scheme

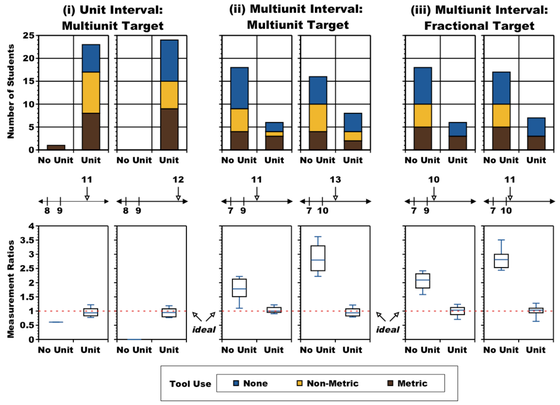
- **Strategy:** Whether students iterated and/or partitioned the marked interval in order to place the target number at an approximately appropriate distance from the marked interval.




- **Tool:** Whether students used 'tools' to mediate their placements:
  - **None:** No evidence of tool use
  - **Non-metric:** Gestural activity, such as pulsing
  - **Metric:** Rigid object used as measure (e.g., pen cap)
- **Accuracy:** A ratio (a/ideal) between the distance from the rightmost tick mark to the student's tick mark for the target number (a) and the ideal measurement .




## Results




- **Strategies:**
  - Students tended to conflate marked intervals with the idea of a linear unit.
  - Significant effect of item type on the use of a unit strategy, $p$ <.001. Students were more likely to use an appropriate strategy on the unit interval tasks than on either the multiunit - multiunit target tasks, $p$ <.001, or the multiunit - fractional target tasks, $p$ <.001.
  - No difference in performance on multiunit tasks.
- **Tool Use:** Students' use of tools varied independently of their use of a linear unit strategy.
- **Accuracy:**
  - **Unit strategy used:** Positional accuracy was distributed within a narrow range of the ideal.
  - **Non-unit strategy used:** Positional accuracy was

## Educational Implications

- Students' successful performance on routine number line tasks may not reveal their difficulty coordinating numerical and linear units.
- A measurement context (e.g., a race course) may support students' insights about relations between linear and numerical units.
- Non-routine number line and measurement tasks have potential to support and assess students' engagement with important mathematical ideas.

*This research was funded by a grant from the Institute of Education Sciences (#R305B070299), "Teaching Fractions and Integers: The Development of a Research-Based Instructional Practice"