What Constitutes Evidence of Teachers' Learning from Lesson Study?[1]

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Although "Lesson Study" is rapidly spreading across the US, there has been little public conversation about how teachers learn during lesson study, or what constitutes credible and useful evidence of lesson study's impact on teachers. In order to help support such a conversation, we briefly present a framework for studying teachers' learning during lesson study, and then ask audience and discussants in this interactive symposium to react to three pieces of evidence drawn from sites where lesson study is emerging in the U.S.

Background of Today's Presentation

Figure 1 presents a framework for studying how teachers learn during lesson study, based closely on Deborah Ball's (2001) model of teachers' learning from practice as they interact with three types of resources: other teachers, students, and content (see also National Research Council, 2001). Essentially, we situated lesson study within the model of learning from practice that Ball has proposed. At the base of the framework we added a box describing the learning capacity built through lesson study over time; this learning capacity is conceived as both an outcome of lesson study and an input to further development of lesson study. A detailed discussion of this framework, including audience comments from last year, can be found at www.lessonresearch.net under AERA 2004.

Last year we solicited written feedback from audience members on the framework and on three pieces of evidence of teachers' learning during lesson study. Building on the comments of audience members and discussants last year, we have modified the model and will present three new pieces of evidence this year, again asking audience members to respond to the evidence in writing. The evidence is intended to highlight some of the challenges we face in documenting teachers' learning during lesson study.

Modifications to the Model

Figure 2 highlights the major change in the framework from last year to this year, which occurred within the box titled "learning capacity" (formerly titled "capacity") at the bottom of Figure 1. Last year, several audience members noted the importance of seeing how the knowledge gained in lesson study fares over time; as one audience member asked: "How does [teachers'] "new" knowledge react to new situations?" The

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box now elaborates three sets of processes: (1) acquiring knowledge; (2) generating connections among old and new knowledge and reorganizing, elaborating, and pruning knowledge so that it becomes increasingly coherent and powerful; and (3) self-monitoring progress; these processes are based on the "knowledge integration" model developed by Marcia Linn and colleagues (for recent descriptions, see Linn, Eylon, & Davis, 2004; Linn & Hsi, 2000). Like the "knowledge integration environments" that enable science learners to forge increasingly coherent, powerful scientific understanding from their disconnected and sometimes contradictory knowledge (Clark & Linn, 2003; Linn, Eylon, & Davis, 2004; Linn & Hsi, 2000), lesson study may be regarded as a knowledge integration environment where teachers may have the opportunity to build increasingly effective, coherent knowledge for teaching through acquisition of new ideas, connection of new and old ideas, and refinement of fragmented and inconsistent ideas.

Last year, we included the words motivation and efficacy in the capacity box, to capture our thinking about the qualities that enable continued building of capacity. We now include motivation and sense of efficacy among the components of knowledge that may be strengthened or discarded over time as they bump up against other knowledge from practice, formal learning, or other experiences (Linn, Eylon, & Davis, 2004; Linn & Hsi, 2000). For example, the comment of one of the teachers in today's video that "It's kind of fun to think about all the different things you can … tweak [in the lesson] and then … watch and see what …[students] do" represents an idea about the possibility of improving instruction that may be strengthened or discarded over time.

A second modification of the framework is to include "research feedback" in the professional resources circle, in order to highlight use of research artifacts that capture lesson study work in some way (for example, through video, written cases, researcher notes, transcripts), making the lesson study process itself more visible to participants. Sparked in part by the recent lively discussions about design-based research (Barab & Squire, 2004; Collins, Joseph, & Bielaczyc, 2004; Kelly, 2003), our attention to research feedback relates to *why* researchers gather evidence of teachers' learning during lesson study. We see three major reasons to investigate teachers' learning during lesson study

1. **To develop a richer set of hypotheses about lesson study: how it works, the pathways of impact on teachers, etc..**
   Given that knowledge of lesson study is in its infancy in the U.S., we see a critical need to gather evidence that helps generate a knowledge base about how teachers learn during lesson study (and from practice more generally), in order to develop productive hypotheses and models. Like the knowledge base for teaching itself, however (Hiebert, 2002, Hiebert, Gallimore, & Stigler, 2002), the knowledge base for lesson study may not be readily captured in forms most familiar to researchers, and may require varied and innovative representations (e.g., public research lessons, multimedia cases).

2. **To conduct design-based improvement of lesson study.**
   Second, evidence of teachers' learning can be used to identify the specific features of lesson study that support learning, and these features can be systematically refined and studied in design-based research. The examples

presented today point up particular lesson study features (such as close observation of students, study of existing curricular materials, and inclusion of outside content specialists) that may be important design features.

3. **To determine whether lesson study has an impact on instructional improvement and on student learning.** In an era of unprecedented emphasis on accountability and on research-based evidence (Eisenhart & Towne, 2003; Shavelson & Towne, 2002), there is widespread interest in knowing whether lesson study has an impact, measured in scientifically credible ways. Some members of last year's audience suggested that evidence of teacher learning is merely a proxy for more important outcomes, notably changes in instruction and in student learning. In response to last year's prompt "What additional types of evidence would convince you that teachers are learning through lesson study?" audience members wrote:

   "Seeing changes in their actual teaching practice"

   "Teacher learning must result in improved student learning outcomes if it is to be powerful and worth the effort/time of lesson study;"

   "It has to be connected to achievement."

   "The only learning that is valuable is that which produces increased student learning. Therefore, evidence that this type of professional development leads to student learning with greater gains than other types of professional development would be useful."

Although we view all three purposes for evidence of teacher learning as important, we see the first two purposes – developing a broad knowledge base about how teachers may learn during lesson study and identifying the specific lesson study design features that are important -- as logically prior to purpose three, as illustrated in Figure 3. Lesson study has a shallow history in the U.S and there is little reason to think that the key pathways for teacher learning during lesson study have already been identified and incorporated into US lesson study practice (Lewis, 1999; Lewis 2002a,b; Lewis, Perry, & Hurd, 2004; Fernandez, Cannon & Chokshi, 2003). Hence, development and refinement of lesson study needs to occur before summative study is likely to represent a reasonable investment.

We recognize that many researchers might argue for the opposite order, asking for evidence of lesson study's impact *before* generating a knowledge base about how teachers learn during lesson study or what design features of lesson study are important. However, two considerations convince us that knowledge-building should take priority over summative research. First, Japanese practice already provides an existence proof of lesson study's potential to reshape instruction over time in powerful ways, given the right supports (Lewis & Tsuchida, 1997; Stigler & Hiebert, 1999). Second, the English-language knowledge base on lesson study is extremely limited; only two cases of the complete Japanese lesson study cycle are available in English and nearly all US cases have all evolved from these, although Japanese lesson study is an extremely variable practice that has evolved over a century in tens of thousands of

Japanese sites. If we think about characterizing any complex U.S. practice (such as cooperative learning or whole-school change) on the basis of just two cases, the limitations of our knowledge base become clear. The knowledge needed to build credible models of lesson study worth researching should not be underestimated, particularly given the inadequately developed infrastructure for educational R&D (Burkhardt & Schoenfeld, 2003).

Lenses for Examining the Evidence

In order to focus on challenges in lesson study research, we ask audience members to consider three questions about the evidence that we present.

- How important is the knowledge pursued by the teachers?
- How did features of lesson study support teachers' learning?
- How can we predict whether the teachers' learning will be generative – that is, whether it will lead to continued learning and improvement of practice over time?

We briefly explore the rationale behind each question.

Question 1: Is the knowledge important to teaching?

A broad range and great depth of knowledge about students, subject matter, and pedagogy are needed for teaching (Shulman, 1987; Lampert, 2001; Ball & Bass, 2000). Indeed, one Japanese teachers' association calls itself the "Polar Exploration Group," in order to call attention to the extensive knowledge needed by both teachers and explorers to function in chronically unpredictable environments. Within mathematics alone, teachers need many types of mathematical knowledge, some of it distinct from that needed by other professional users of mathematics such as engineers and mathematicians (Ball, 2003; Ball & Bass, 2000; Ma, 1999).

When we present evidence of teachers' knowledge development like that which follows, we often encounter disagreement about how important the particular knowledge is. For example, for the problem "How Many Seats?" (see video), last year we showed video evidence of one teacher's shift from not understanding the meaning of the "plus two" pattern in the table to being able to explain that the number of seats is two more than the number of tables. How important is such a shift? While we did not explicitly ask last year about the *importance* of this knowledge, many audience members commented on it as compelling evidence of knowledge development, while at least one audience member described this knowledge as "trivial." How does being able to explain the plus two pattern compare in importance with the same teacher's subsequent understanding (shown in today's video) that the plus two pattern connects to geometric characteristics of the problem (i.e., that the two ends each contribute an "extra" seat in addition to the single seat contributed by each triangle)?

A similar difference of opinion often emerges in response to a video segment "rule, formula, equation"(not shown today) in which elementary teachers writing their research lesson plan encounter different ideas among group members about the meaning of the terms rule, formula, and equation. They consult sources including the textbook and middle-school mathematics teachers. Audiences have widely differing reactions to this segment, with some seeing the work to accurately define mathematical vocabulary as core to mathematical knowledge development and others seeing it as unimportant (e.g. "rule" is not a mathematical term) or not a good use of time (e.g., texts with clear definitions should be available). Delineations of the big ideas in mathematics may provide some guidance (e.g., National Research Council, 2001), but the relative importance of the many kinds of knowledge needed by teachers, and the order in which they are most easily built, remain big questions.

Question 2. How did features of lesson study support teachers' learning?

Concrete examples of teachers' learning during lesson study may be critically important resources for US sites seeking to build lesson study, since it may be far easier to re-create in one's own setting a particular pathway (such as anticipating student responses) than to imagine that pathway from scratch. A chronic reason for reform failure in the US has been implementation of the visible features of reforms without full grasp of their underlying principles (Fullan, 1991; Sarason, 1990; Spillane, 2000); evidence useful to the design of lesson study might capture these underlying principles and their connections to design features in ways that enable other sites to understand them and create effective adaptations rather than "lethal mutations."

3. How can we predict whether the teachers' learning will be generative – that is, whether it will lead to continued learning and improvement of practice?

Over time, does the knowledge gained in lesson study disappear in the crucible of practice, or does it lead to increasingly coherent, effective practice? We can all think of things that we once knew but no longer know. Teachers' knowledge revealed in a snapshot at a particular point in time may be fleeting, or it may be centrally important to future knowledge development. What predicts the two different fates?

Written Comments on the Evidence

The evidence is provided in cases 1-3 (and on video). Please comment on it in writing using the forms in Appendix A.

Closing Thoughts

Researchers seeking to provide useful, scientifically credible evidence of teachers' learning during lesson study (and other forms of practice-based professional development) face difficult dilemmas.

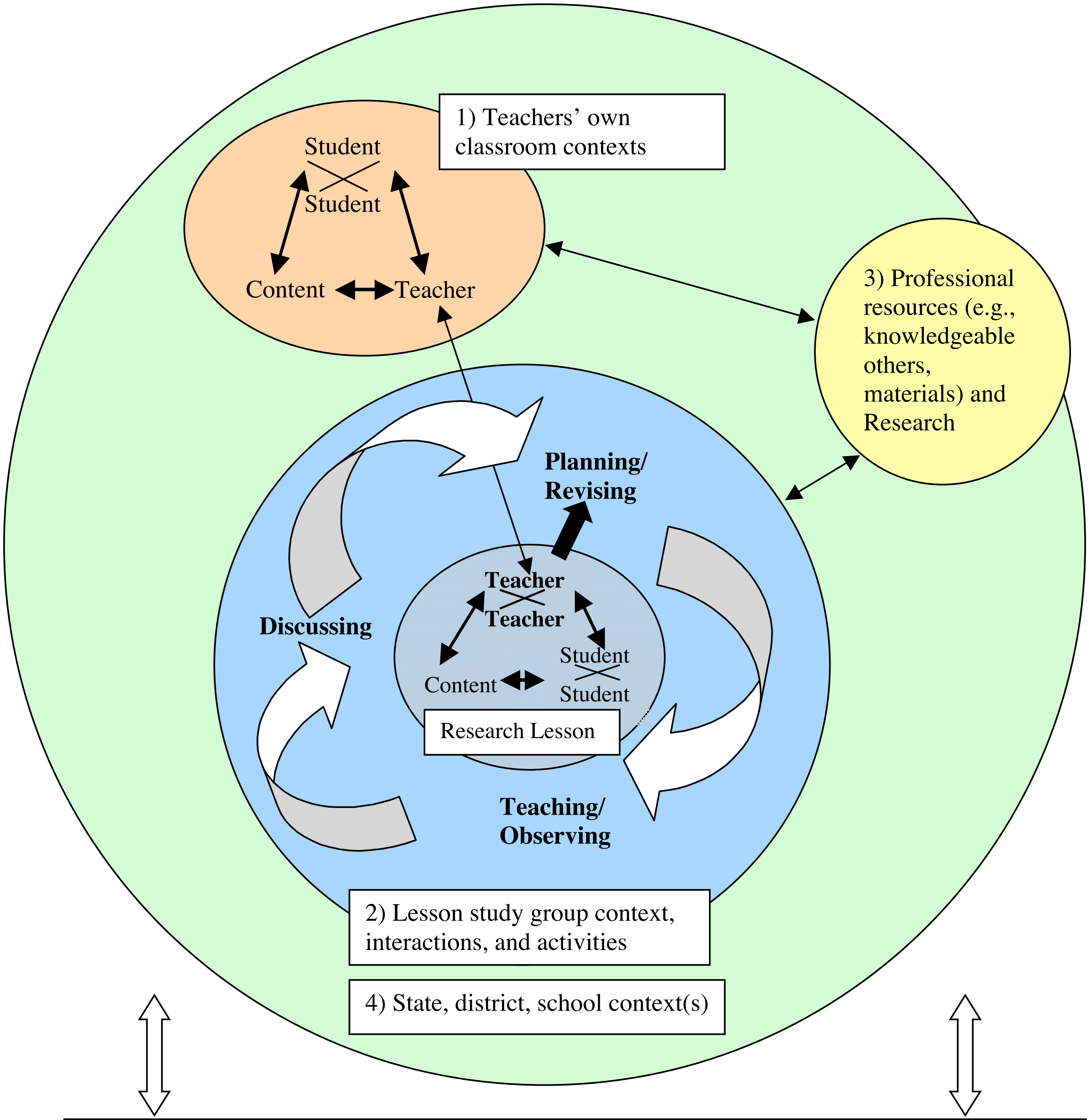
1. **Proving vs. Improving.** Do we focus on determining whether lesson study has an impact on student learning, or on generating a much broader base of hypotheses about how lesson study might support teachers' learning? Perhaps small-scale documentation of the classrooms where lesson study is occurring – like the study of students' understanding of the "seats" problem in response to the first and second teachings of the lesson – can provide something of a middle ground between the two goals.

2. **Extreme Variability**. Lesson study is highly variable both from group to group (since group members choose content) and within groups (since no two teachers enter with exactly the same questions and needs). For example, individual group members within the "How Many Seats?" video each reported learning

something different from the experience. Lesson study's adaptability to local needs and differentiation to practitioner's needs is probably the key to its rapid, grassroots spread, since many educators see it as a means to address the problems they confront in their own classrooms (Lewis, 2002a,b). Yet policy-makers need to know what teachers *generally* learn from lesson study, and so researchers must look for patterns of learning across variable local cases. Looking across the three brief pieces of evidence provided today, for example, we might conjecture that lesson study should enable teachers to understand the particular ideas they pursue -- such as "solve in multiple ways," "symmetry" and "understanding of patterns" -- in more rigorous, practice-grounded, instructionally productive ways.

3. **Useful Knowledge for Lesson Study Practice, Research, and Education Reform.** Different forms of knowledge are differentially useful to lesson study practitioners and researchers. For example, today's edited video may be more useful to lesson study practitioners, for its concrete images of teachers' learning pathways, than to researchers (for whom an unedited version is probably preferable). On the other hand, complete discussion transcripts that capture the changes in thinking of all group members and the role of outside content experts may be much more useful to lesson study researchers than to practitioners. As researchers concerned with the improvement of education, how do we accurately grasp the needs of both practitioners and researchers with respect to evidence about learning during lesson study, and how do we make thoughtful decisions about whose needs to target at this critical early stage of lesson study's US history?

Figure 1: Development of Learning Capacity Through Lesson Study



Learning Capacity: * (1) **Acquiring Knowledge** for teaching** and for lesson study, attitudes toward students, subject matter, and colleagues (2) **Connecting Knowledge**; (3) **Self-Monitoring Progress**

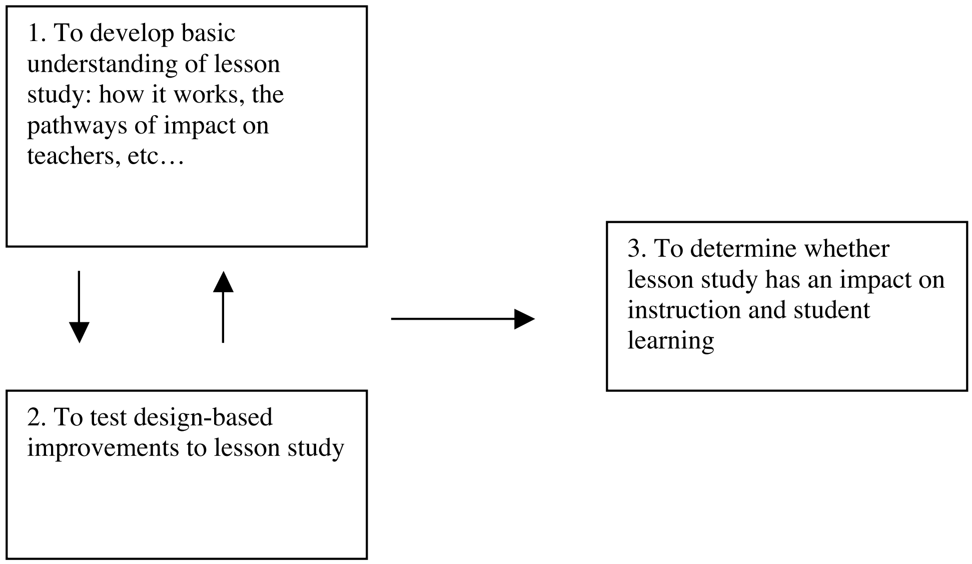
Graphic adapted from Ball, D. (2001). Studying practice to learn in and from experience. Invited keynote address to the California Mathematics Council annual meeting, Asilomar, November, 2001.

*Learning model based on knowledge integration environment described by Linn, Eylon, & Davis, in press

Figure 2. Change in Framework

AERA 2003	**Capacity:** knowledge for teaching;* knowledge of lesson study; motivation/ efficacy to put new knowledge into practice
AERA 2004	**Learning Capacity:**(1) **Acquiring Knowledge** for teaching and for lesson study, attitudes toward students, subject matter, and colleagues) (2) **Connecting Knowledge**; (3) **Self-Monitoring Progress**

Figure 3. Reasons to Gather Evidence of Teacher Learning During Lesson Study



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Example 1 - Understanding Student Thinking of "Solving in Different Ways"
Case of Primary Grade Lesson Study Group
(Aki Murata)

(Topic: Combining story problem with unknown change)

Background:

The lesson study group consisted of 4 teachers who worked at a K – 5 elementary school in Western region of the United States (2 Grade 1 teachers, 1 Grade 2 teacher, and 1 Special Education teacher), plus 2 outside collaborating educators. May (pseudonym) was one of the Grade 1 teachers in the group, and this was her third year with the district's lesson study effort. May had taught in the same school for the past 6 years and knew other members of the group very well. She has worked with two of the teachers in the lesson study group before. This year, the group decided to focus on the topic of combining story problems with unknown change since another Grade 1 teacher (Shelley, also pseudonym) identified that it was a difficult topic to teach. The group met approximately once a month during the school year, starting in October, and the research lesson was taught in February, and revised and re-taught in March. Data were gathered as digital audio recording as well as videotape recording during lesson study meetings and research lesson. These were transcribed later for analysis. Lesson artifacts (e.g., multiple versions of lesson plan, worksheet) were also collected and analyzed.

Nature of teacher knowledge developed:

May came to a new understanding of what it means to "solve a problem in different ways." She realized that when her students were showing thinking in different ways (with pictures, numbers, and words), they were not necessarily solving problems using different kinds of thinking. May set as her future goal paying more attention to students' thinking as they worked on problems, after realizing the limitations of what she could understand from the finished papers.

Features of LS that afforded the knowledge development:

Group discussion, exchanging of ideas, constant reflection, seeing actual student examples, data collection sheet, comments of math specialist (collaborating educator)

Evidence of teachers' learning drawn from collected data is presented in the following pages, with additional interpretation of the data added by the primary researcher:

Date	Evidence of teacher development	*Researcher Interpretation*
Oct	In lessons, May asks her students to solve a problems in "different ways." "What different ways can we solve problems?" Students answer "Pictures, numbers, or words!"	*May clearly communicates expectations that students will solve problems in different ways; students interpret pictures, words, numbers as different.*
Dec	May assessed her students informally in the classroom with a story problem with unknown change. Most students showed their thinking in multiple ways (same thinking in multiple ways).	
	In the planning meeting, the group generates a list of anticipated student responses based on in-class student assessment data and discussion of student strategies afterwards: 1. If they know the total, they might plug in different numbers that don't match the problem 2. Some may use subtraction 3. Most will count up by using fingers or a number line 4. They know the combination already and don't really need to think about it 5. Some will add the 2 given numbers together 6. Some will draw pictures 7. Some will use cubes	*Anticipated responses listed are not exclusive of one another, yet teachers discussed them as independent. For example, subtraction (2) may be shown by counting up (3), using pictures (6) or cubes (7).*
Feb	In the planning meeting, the group finalized the first research lesson plan, One of the lesson goals was: Students will develop strategies such as counting on, using cubes, and drawing pictures to help solve story problems that involve combining with unknown change.	*Teachers see "counting on," "using cubes," and "drawing" to be different strategies, although counting-on may be shown by cubes and drawings.*
	For the research lesson, math specialist (collaborating educator) prepared a data collection sheet showing different strategies; drawing and using cubes are shown side by side representing the same strategy.	
	In research lesson debriefing, teachers discussed how strategies students had shared in the lesson were the same or different: *Carol*: Although they have all written differently, in my opinion, they were the same … For them to explain why they are different, the concept of things … *May*: If the kids were trying to elicit the differences (in the discussion), they might have then shared that they had drawn all ten and then drawn the line … (if) you are wanting them to share why it's different … (they might have explained as) by drawing seven and then three more.	*As Carol shared her observation that she saw the strategies shared were the same, only written differently, May said that if students were asked to talk about differences, they might have explained the ways they had drawn the pictures, and the strategies might have indeed been different. The discussion did not extend far enough and there is not enough evidence to talk about differences among strategies shared.*

	In research lesson debriefing: *Shelley*: If I am working with you over here, and (students) do circles and add, I am not going to know that they did what … *Cindy*: … they try to come up with more than one way on the paper .. *May*: … kids did it in more than one way, and they say, oh, I wrote the number, write a sentence, drew the picture, but then what we are saying is that I start with ten and then I take away seven or eight … all they are doing things on the paper the same way but showing it in different ways like they, like show different methods …	*May is making a distinction that while a student wrote numbers, wrote a sentence, and drew a picture, he/she might be showing the same thinking process in multiple ways.*
	After the research lesson, May wrote an email message to another collaborating educator: *" WE had a great debriefing session today! Partly because we are more experienced and partly because (math specialist) created this document that outlined the anticipated responses. We recorded our observations on the sheet, our data then served as a springboard for our discussion. Coming out of this session I realized that the anticipated responses made us focus back on our lesson goals. It also provided us with clear data to help revise the lesson. I've never felt this confident about teaching a lesson in front of a group in my life …"*	
Mar	In research lesson debriefing: *Shelley*: I saw your kids did kind of what my kids did in terms of showing in different ways, too … even though they solved it one way, they said they did the equation … which is interesting, I don't know why both of the classes thought they had to do more than one way, but they did. *May*: And lots of pictures, numbers, and words, *Shelley*: They want to do a picture, number, and word, *Mary*: And I thought that was priceless when one little girl said that they used pictures, numbers, words, (another student said) you took the words by my mouth! *Shelley*: Yeah, that was so cute.	*The classroom expectation of encouraging students to use "pictures, numbers, and words" is reflected on student work, while teachers seem not to know why students used those three ways to show their thinking.*
Mar	In research lesson debriefing: *May*: … this is a goal of mine, to have a better plan (for) I was going to share because as you could see, I was like struggling at the end to just grab someone to share and that's not really good. I did not know who did what, could not tell from their papers … when (I wanted the discussion to be more like) you look at these papers more and explain a lot in the conversation … so I still have lots of goals.	*May expresses her intention to be more observing of the process of student thinking in the future and plan better in order to know whose work to choose for a discussion beforehand. She realizes that finished papers do not always show student thinking process.*
Mar	In the written group reflection form: *"Very important to be able to "show how" you did a problem, not just say, "I solved the problem," showing and explaining how is VERY important. The process is more important than the answer."*	*The group reflects and discusses the importance of thinking process to get to an answer.*

Example 2 - Teachers' Learning about Line Symmetry (Rebecca Perry)

This case focuses on a group of teachers engaging in lesson study during a two-week summer workshop devoted to lesson study and transformational geometry. The group consisted of five grade 3-6 teachers from three local school districts in the San Francisco Bay Area. This case highlights learning during only one cycle (three days)[2] of lesson study work at the beginning of the workshop, with additional evidence (from the revised research lesson taught the next week, and teachers' reflections immediately and seven months after the workshop) also included to illustrate teachers' thinking after the case study period.[3] The topic of focus for the group was line symmetry[4] taught to a fourth grade class.

Data sources for the case study included: Videotaped and partially transcribed lesson planning, classroom, and debriefing sessions; written artifacts (e.g., lesson plans and rationale, daily notes, teacher reflections); and emailed responses to a researcher question about learning. The research methodology involved reviewing videotapes and artifacts to identify important themes in the teacher discussion. As themes were identified, the data were reviewed multiple times to understand and document the evolution of ideas discussed within the group.

This case highlights two primary areas of teacher learning about symmetry: 1) Understanding the value of proof and justification of ideas as a mathematical "habit of mind;" and 2) A definition of line symmetry that includes a formal (mathematical) definition and the idea of defining the attributes of symmetry according to a given circumstance. Additional learning that seems to have occurred, but will not be a primary focus here includes: how certain tools (e.g., pattern blocks) and pedagogical strategies might support or challenge students' reasoning about symmetry; and how mathematical ideas about symmetry connect and build on each other.

Several features of lesson study played a role in teachers' learning:

- Time to hear, revisit, and reflect on ideas;
- Pressure to teach the lesson on the third day;
- Teachers' review of the Navigations curriculum and standards (regarding symmetry) provided by workshop leaders;
- Opportunity to hear about student understandings across grade levels;
- Teachers' willingness to talk about their own understandings of symmetry (trust, individual initiative);
- Culture of active learning within the group (e.g., questions, constructive criticism, corrections, exploration, acknowledgement of not knowing and learning are all acceptable);
- Careful observation of students during the lesson in light of stated lesson goals, for the purpose of informing further discussions about symmetry;
- Collaborating outside math specialists inclined to ask questions that elicit discussion and questioning among teachers rather than solely providing answers.

These features of lesson study combined with each teacher's own knowledge, attitudes, and skills brought to the collaborative work, including:

- Prior lesson study experiences;
- Use and knowledge of their own district curriculum about symmetry;
- Knowledge of understandings and misconceptions for students at their grade level;
- Mathematical curiosity and interest.

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[2] The three days included one 2 hour planning period on the first day; one 3 1/2 hour planning period on the second day; and a one hour classroom lesson, a one hour debriefing session, and one hour of planning on the third day.

[3] The data collected during the case period (7/29/03-7/31/03) are presented above the triple line in the table; follow-up data after the case study period are presented below the triple line.

[4] Throughout the remainder of this case description, the term "symmetry" will be used to mean line symmetry only (e.g., excluding rotational symmetry).

Evidence of teachers' learning drawn from collected data is presented in the following pages, with additional interpretation of the data added by the primary researcher.

Date, Tape #, Timecode	**Evidence of Teacher Development**	**Researcher Interpretation**
7/29, 1, 3:25 7/29, 1, 19:05 [From Navigations curriculum]	G3[5]: [Describing a 3rd grade symmetry lesson she has taught.] "It was taking a mirror and putting it on a pattern that they made, half a pattern. And then they would take the mirror and put it on this half and see the reflection." G3: "In third grade, it's [symmetry] basically defined in a sense of if it's the same on one side [in a mirror reflection] then it's the same on the other side." Symmetry: "A line that divides a figure into two halves such that the halves are mirror images of each other."	Teachers' understanding of symmetry is drawn from their classroom experience or available curriculum materials and embedded in student activity. Teachers compare ideas about symmetry offered by each other and the given curriculum. Symmetry is informally defined within the group as "the same" or "mirror image."
7/29, 1, 17:00-17:20	B5: "What helps me a lot is taking it out of the book and folding it and looking at the disparity between what I've drawn and what it should be. Like folding it. Even yesterday when we were doing an activity, I think I was the only person developmentally who needed to fold it, to double check myself. "	The idea of "double checking" or proving symmetry is introduced. B5 describes her own strategy for finding symmetry, which involves physical manipulation (folding). Teachers are able to compare their own strategies for finding symmetry with another adult's and what they know about how children find symmetry. The group agrees that physical manipulation is important proof.
7/29 1, 37:20-37:59 7/30, 1, 10:00	B5: [Re: a 10 pattern block design] "We know that you have to have 5 pairs in order to have the symmetry." [Another teacher disagrees.] "I mean, you have to have pairs. It's gotta be 5 of something and… No?" [Another teacher demonstrates for her how a single pattern block shape can have symmetry.] "Oh, right, right, right." G4: [Describing a 4th grade symmetry lesson] "Kids are asked to draw a line... The first partner lays the pattern block on one side touching the line and the next person does the same. So they are mirror images of one another. And I believe that that is how the term gets introduced… B5: "I think having taught this unit, I think this is why I never considered the odd number of blocks a possibility."	One teacher exposes her own misconception about symmetry (that 1:1 correspondence of pattern blocks on each side of the line of symmetry is necessary for symmetry). The group discusses the symmetry attribute of "evenness," enabling them to refine their definition of symmetry by understanding what it is *not.*

5 Teachers' names are replaced with alphanumeric characters including a letter and the teachers' grade level.

7/29, 1, 22:00-22:17 7/29, 1, 22:28-23:04	G4: "When they go to take assessments [she gives the SAT-9 example], there were some symmetry problems... And the kids could not fold the paper or cut it up or do anything." C6: "If we're talking about line symmetry, we need to define what does the line of symmetry mean. If it's been defined as an activity, you can fold it and it matches, then…" G4: "…then it's not fully defined." C6: "If it's defined as hold a mirror up to it, then the next step, I think, is to come up with a more mathematical definition of the line of symmetry…without the activity."	Teachers discuss the need for students to be able to move beyond an activity-based definition of symmetry toward being able to visualize and prove where a line of symmetry falls.
7/29, 1, 25-26, 26:30-28:10 7/29, 1, 52:00 7/30, 1, 29:17-32:40	D3: [While examining one assymetrical figure] "The distance from the vertex on the left side of the line is not the same as the distance from the vertex on the right side of the line." B5: "We need to keep in mind the fact that points are equidistant from the line."	After C6 expresses the need for a more mathematical definition, one is offered by an outside math specialist. His definition refers to corresponding vertices on either side of the line of symmetry. Teachers elaborate on and continue to refer to this definition later in their planning discussions.
7/30, 1, 29:10-29:42	D3: "The line from which all the points are equidistant is maybe not what we want the kids saying, but something we need to keep in mind ourselves of what symmetry is. And we need to keep that as our ultimate end focus. Maybe not today, but that's where we're pushing them to."	Teachers consider the more formal definition in light of what they know about students from their discussions, prior experiences, and review of CA and NCTM standards. They agree it is not appropriate to have students articulate this kind of formal definition at the beginning of the 4th grade school year, but feel that they should be aware of this knowledge to enable students to build toward this.
7/29, 2, 26:30-27:13 7/29, 2, 28:09-28:44 7/29 2, 37:23-37:48	B5: [In response to a question whether a multi-colored design has one or multiple lines of symmetry.] "You have to include the color…. It never occurred to me not to include the color." D3: "Really!… I don't know that I would consider it, so I think that's interesting." B5: "…This has been a really great discussion because pattern blocks are a staple in our room and it's true that the color and the shape are one in the same. And therein lies the problem. When you use these blocks…" G4: "The children just see it as one concept, but it's actually two."	The group discusses how students' consideration of pattern block colors in a design could influence their understanding about symmetry. They discuss the distinction between symmetrical shape (outline) and symmetrical appearance (component parts). They also discuss how the use of pattern blocks might create confusion for students. Both color and appearance continued to be discussed throughout the two-week workshop.

7/30, 2, 29:17-30:15 7/30, 2, 29:50-30:06	G4: [Stating the definition they wanted students to be able to articulate about symmetry]: “The sameness of size and shape on both sides of the line of symmetry.” D3: “That [the definition] doesn’t address the equidistance… I could have a square and my line of symmetry and a square [shows second square in different position]. And it’s the same size and it’s the same shape but it’s not… position. It says nothing about position.”	After hearing this definition, a second outside math specialist encourages the group to develop a precise definition, explaining that a design that had “sameness on both sides” could be congruent on both sides of a line of symmetry, but still not be symmetrical. This discussion highlighted a third attribute of the symmetry definition (orientation, or position across the line).
7/30, 2, 45:19-45:16 7/30, 2, 46:44-47:20 7/30, 2, 49:35-49:50	B5: “I have a math question… Can this [a design made of two green triangles and a blue rhombus] be symmetrical and not symmetrical? Is symmetry only dealing with the shape of an object or also its appearance? I’m actually not sure… Maybe it’s both and you define it.” B5 [paraphrasing the outside math specialist]: “One of the things that we want to teach them is that symmetry… You have to define the parameters…”	Teachers revisit the attributes of both color and appearance in their planning, suggesting they are continuing to think about these attributes of symmetry. An outside math specialist answers B5’s direct question: “You define the domain of what it is. Does the domain include color? Does it include these little lines in between when you put two things together? Once you define what it is, it’s either symmetrical according to line of symmetry or it’s not…. But you have to define what it is.” The group realizes that the line of symmetry concept is flexible, depending on how symmetry is being defined.
7/31 Lesson plan	The unit goal in the lesson reads: “students will be able to recognize lines of symmetry without folding or using mirrors. They will have mental tools or know aids so they can complete designs symmetrically.” The lesson goal reads: “Creating designs with symmetry and identify lines of symmetry in their designs. Students will recognize that line symmetry doesn’t require matched pairs. Additional text within the lesson plan states: “Students will notice symmetry in the design and describe it as ‘same on both sides, equal, if you move it to the other side, if you fold it the sides will match exactly, it has a line of symmetry, it is symmetrical.’ We’re looking for the following responses [from students during the opening activity] in order to move on: same color, shape, and position on each side of the line.”	Teachers’ lesson goals after 5 1/2 hours of planning reflect understanding about the multiple attributes of symmetry. Their lesson plan reflects what definitions they anticipate from students and how they will assess students’ understanding of symmetry.

7/31 debriefing, lines 399-427	B5; [During planning] I… had made the comment that it has to be matched pairs in order to be symmetrical…And Teacher C6 goes [shakes head no]. And I go "what do you mean?" …Just like that little girl on the carpet "you can't cut…" I mean, it never occurred to me that you could do that. I was like "whoa!"… I think we felt like if we had started with four blocks then they never would have been pushed. We would have had this matched pair kind of a thing and we might have… felt like "OK, they got it." But they wouldn't have gotten what I got when Teacher C6 says "no, it's not just matched pairs." And we wanted to give them that.	During the lesson a student says that a 3-block design with a trapezoid in the middle cannot be symmetrical because "if you cut it in half, it's no longer a shape." B5 relates this student's misconception to her own learning about 1:1 correspondence.
Debriefing, lines 167-173 7/31, 3, 32:30-33:12 7/31, 3, 41:07-41:26	C6: "The student to student articulation [of the definition of symmetry], which we wanted to look at to see if it would help in their understanding of symmetry…, it has already been referred to that for many kids it didn't occur. And I think one thing we were all hoping for is that kids would be able to refer back to that definition. When they looked at the coffee stirrer [the physical representation of the line of symmetry], would they be able to say "is it the same blocks on this side of the line and this side of the line?" "Is it the same number of blocks on this side of the line and this side of the line?" And "are those blocks in the same position on this side of the line and this side of the line?" And I don't know if emphasizing more of that when we do our definition would help encourage that checking or that proving. What I saw was "can you fold it in half?" Or just sort of a simple or cursory "look!" "Oh, OK." "Look!" "Oh, OK." And maybe that's good enough at this time of the year for fourth graders and maybe that's something that we want to adjust in the revision." G3: "The confusion of the parameters, as far as shape, color, and positioning… I wonder if it would help perhaps if you had a sample of each different one because some of the kids were confused… you could tell that they were going 'ok, is it color…?'" B5 agreed "The attributes about what makes a shape symmetrical didn't get discussed."	Teachers observe that the definition of symmetry that they were hoping for from students was not elicited during the lesson. While teachers' had developed their own understanding of symmetry, they found that their definition was, in this case, beyond the students' knowledge. This led them to discuss whether the definition needed to be adapted or whether the lesson did not enable students to understand and/ or articulate the attributes of symmetry.

7/31, 4, 10:45-11:12	B5: "Is color going to be one of the criteria for symmetry in this [lesson]…? What *are* we saying is the definition of symmetry? Are we going to explain or talk about that there are different parameters and for this particular lesson yes it [color] is [part of the definition] or leave it? I felt like it was left unstated today. And so the kids couldn't really walk out the door and say "Yeah, color matters" or "No, color doesn't matter."	Teachers saw that the color attribute of symmetry was not a concern in the first lesson for students. They found their lesson's use of pattern blocks did not elicit any discussion about color, but were not convinced that this meant students understood symmetry.
7/31, 4, 16:45-17:20	B5: "In fourth grade, there is a science unit that deals with symmetry. And one of the extensions is to go around and find it. And they would say this [8.5 x 11 inch sheet of] paper has symmetry. And I'd say this [blank side of the paper] is symmetrical, but is this [side with text on it]? If I fold it, am I going to be able to put a mirror on this paper and see the same thing? Not realizing it then, but what I was pointing out was we have to define the parameters. Are we talking shape – yeah [it is symmetrical]. If we're talking appearance, no [it is not symmetrical].	Through the lesson study work, B5 is able to verbalize her understanding about symmetry.
8/7/03 lesson plan and rationale	The revised lesson plan goal reads: "the students will begin to define 'symmetry' in a shape or design as the object having the same shape, same color, and same position across a line." The written rationale for this revised goal reads: "We wanted our lesson goal to more explicitly state what criteria students will be encouraged to focus on in their analysis of both a shape and a design." Changes to the revised lesson included: -- The idea of a "hatsumon," or focusing question for the lesson, was introduced: "how do we know when a shape or design has symmetry?" -- Emphasizing attributes of symmetry during the opening activity (e.g., by the teacher eliciting a student's meaning when he said two halves of a design "match"). -- The class explored symmetry in individual shapes to eliminate the misconception of "evenness" that arose in the first lesson and understand that it is possible to divide single shapes in half. -- Students were allowed to fold paper to prove symmetry. -- At the end of the lesson, student designs were categorized into symmetrical and assymetrical designs according to the class definition.	Teachers seem to "own" the fact that there are multiple attributes of symmetry that help one to determine if the design or shape is symmetrical. They also realize that students at this grade level may still need to physically manipulate paper to prove symmetry, but not all students will need this.

8/7, 2, 13:35	C6: [Commenting on the second lesson] "Trying to get a class definition for symmetry was pretty tough. We were looking for three things – we were looking for the same color, same shape and hopefully same position across a line. And what we got was 'it would be the same on each side.' And what that makes me think and thought at the time was that when kids said "it's the same on each side," that covers it all. And that they're not able at this time to differentiate between the color and the shape and the position across a line. Towards the end… I was looking for a better example [of student work] to show that the position makes a difference, but that would have been a much longer discussion and [because of time] I didn't want to get into that.	While teachers understand the attributes of line symmetry, they observe that students don't seem to make the distinction between the three attributes. However, they also realize that their in-the-moment pedagogical decisions might help students understand the attributes.
Emailed reflection to other group members immediately following workshop	C6: [Commenting on a major piece of learning for him] "the idea of proof, that is, how do you know that what you believe is correct? And how can you communicate that belief?…" G4: "I will use "how do you prove this has line symmetry?" until they beg me to stop… To get the concept of line symmetry as "position across a line" kids have to take apart shapes to test out their assertions."	Teachers seem to have gained understanding about the value of proof and justification of ideas as a mathematical "habit of mind." G4 reported her intention to use this idea of proof in her own instruction and reflected on pedagogical strategies that elicited particular attributes of symmetry.
7 months later	B5: "One of the recurrent themes for me was the need for more precision. I saw the need for greater precision in our own vocabulary, more precision with the student's word choice and precision with our own and student's work product. That is going to be a real emphasis in my class this year. I learned that the parameters of symmetry have to be defined (at least in the teacher's mind) before students can determine if a design has symmetry. Meaning color and or shape can affect symmetry. I also deepened my understanding of the role of manipulatives, examples, etc."	B5 seems to have "unpacked" the concept of symmetry by learning about its multiple attributes and the need to define "parameters" according to a given circumstance. She has also grappled with the notion that certain tools (e.g., pattern blocks) might support or challenge students' reasoning about this mathematical idea.
7 months later	D3: [Commenting on a symmetry lesson taught by her partner teacher] "[it] could have been better with more emphasis on justifying why designs worked or didn't work and clarification of what makes a design symmetrical." She continued: "…the element of points being at opposite locations across a line – the big idea about symmetry I took away from this summer. I think for my future work with geometry, I'm realizing the importance of being aware of the building blocks of visual-spatial skills, the connections between these skills, and the developmental nature of very similar skills in measurement around unit iteration, and emphasizing logical thinking and justification skills over memorizing terms."	D3's definition of symmetry included a more formal (mathematical) definition. She seemed to be thinking in different ways about the ways that mathematical ideas connect and build on each other.

Conclusion

I hope that this case has demonstrated teachers' learning about line symmetry. But, regardless of the learning that I *believe* is evident from the data, I am left with further questions. First, does this qualify as scientific evidence of learning? Many would argue it does not – it is neither based on an experimental framework nor does it attempt to measure learning at the individual teacher level. As this case demonstrates, lesson study supports different teachers to learn different things – it is by no means a one-size-fits-all professional development approach. Moreover, while it is true that what teachers say about symmetry and what they know about symmetry may not be equivalent, it is clear that the teachers were exposed to a more mathematical definition and arrived at a group definition through their discussion; group members shaped what was discussed and learned. Both of these factors – the simultaneous individual and group nature of learning from lesson study – create complications for researchers attempting to gather scientifically credible evidence of teacher learning. I argue that this kind of study goes a long way toward demonstrating immediate learning outcomes for teachers as they participate within the group, but could be supplemented with additional longitudinal study in teachers' classrooms to understand individual teacher outcomes derived from the group learning process.

Second, is this learning important? I argue that it is. Current federal legislation places significant attention on teachers' knowledge of subject matter. Others argue that teachers' knowledge of mathematics includes their conceptual understanding of the subject and knowledge of appropriate definitions.[6] The case demonstrates that at least one of these teachers articulated weak knowledge about symmetry when the workshop began, and that new knowledge – or at least an awareness of and ability to articulate her knowledge – was built through her lesson study experience.

But a third question arises. Should teachers construct their own knowledge of symmetry through a process like lesson study or be given this information? It is perhaps not a good use of teachers' time to construct mathematical or working definitions for themselves. (Is this not the same question of constructivist teaching that teachers face on a moment-to-moment basis?) Constructivists argue that learning is lasting when students express a need for and construct knowledge for themselves.[7] Similarly, when teachers demonstrate a need (both for the lesson study experience in the first place and, in this case, for a refined definition of symmetry) and engage in a collaborative process of knowledge construction, the knowledge they gain may also be lasting. This case suggests that there is value to supporting teachers' knowledge construction through lesson study.

This case also suggests that lesson study cannot work in isolation. Perhaps one of the reasons that lesson study has a long, successful history in Japan is that teachers' work is supported by the educative national curriculum.[8] Lesson study – and other forms of practice-based professional development – in the US might also benefit from the development of educative curricula that help teachers to understand the mathematics of symmetry across the grade levels in ways that are also useful to their teaching (and do not take the tenor of the mathematics classes that so many elementary teachers avoid in the first place). This case also makes clear that a second helpful support for lesson study could be a system for linking teachers with knowledgeable and understanding mathematics specialists like the two who supported this group by asking good questions.

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[6] National Research Council (2001). Adding it up: Helping children learn mathematics. J. Kilpatrick, J. Swafford, and B. Findell (eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

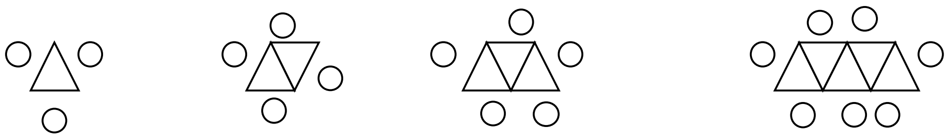
[7] Brooks, J.G. (1990). Teachers and students: Constructivists forging new connections. *Educational Leadership, 47*(5), 68-71.

[8] Lewis, C. & Tsuchida, I. (1998, Winter). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator*, 14-17 & 50-52.

Example 3: Videotape from Summer Institute (Catherine Lewis)[9]

The videotape shows an ad hoc lesson study group of six elementary teachers (from different schools) who worked together to plan, teach, revise, and re-teach a research lesson, during a 2-week summer institute in algebra and lesson study. Their charge was to focus on an aspect of elementary mathematics that provides a foundation for algebra. They studied state standards and existing curricula (including the district's newly-adopted textbook) and decided to focus their research lesson on students' identification and mathematical representation of patterns, using a textbook lesson as the basis for their research lesson. In the lesson, students identify and mathematically represent as a rule the number of seats that fit around a row of triangle tables (see illustration). In the first research lesson, students are given a table (from the textbook lesson) to organize their data (see illustration). After observing that students correctly fill in the table without necessarily being able to describe the pattern in words or an equation, the teachers redesign and re-teach the research lesson, followed by a discussion of the lesson, and then a discussion of what they have learned from the revising and reteaching.

We have a long skinny room and triangle tables that we need to arrange in a row with their edges touching, as shown. Assuming each side can hold one seat, how many seats will 1 table, 2 tables, 3 tables hold? Is there a pattern that helps you figure out how many seats 10 tables will hold?



# Triangle Tables	# Seats
1	3
2	4
3	5
4	
5	

Can you find a "plus-one" and "plus two" pattern? Why do these patterns occur?

Video Transcript	Researcher Notes
FIRST TEACHING AND COLLOQUIUM **Title: First Teaching of Lesson: August 12, 2002** LB: This is what I want you to discover either by yourself or with your partner. What is the pattern. This is what I want you guys to do with your triangles. I want you to fill out to here. Fill out how many seats with three, how many seats with four, with five and with six. Now when you are done with six stop and really look at what patterns do you see on this chart and with your triangles. Alright, and then write about them down here. Any questions, okay, go ahead and get started. Title: All 22 students fill out the worksheet numbers correctly, but only 5 students describe the "plus two" pattern. **Student voices** LB: After you are done writing about the pattern would **you** turn to the person sitting next to you and kind of talk if they are ready to listen and share, see if you guys agree with what you have discovered	

[9] See also videotape materials sent separately.

S1: This is what I got like, every time you go down like three, one plus what equals two. So each time you have to add like two

S2: The pattern is that for the output is two more than the input. Cause once you get that you know the whole thing.

Title: Despite the correct worksheets, few students can explain the pattern in words or express it as an equation

LB: Okay so the number of tables plus two right, equals what. Number of tables plus two that is what? What are we trying to figure out? What are we trying to figure out here? I'm not seeing many hands up.

Title: Lesson Colloquium, August 12, 2002

LB: We wanted the children to investigate the relationship between patterns and rules and come upon that aha themselves. I realized going into this lesson that I wasn't sure how the kids would organize the data and our worksheet set it up for them, kind of spoon-fed them. One of the things we are going to be talking about later is, was the worksheet helpful in focusing their thinking or did it sort of close off that that aspect and not give us the feedback and more about where the students were starting from in this whole process.

JH: And could also generally see that kids were able to fill out the worksheets quickly but never really seen an indication of what does that mean that they know. So I think we all felt a little bit frustrated one it was good to see, oh yes they saw plus two and they could add plus two to all of those numbers. But that that work didn't necessarily show us okay these kids really understand this pattern.

AC: Now the group put together some questions that they wanted to discuss. Did students make a connection that a pattern builds a rule.

LB: At the very end when I was trying when I was trying to get them to say the number of table plus two equals the number of seats there was a lot of confusion. It is easy for them to just go plus two, plus two, plus two and they sort of lose the whole picture of what is the plus two representing.

JH: I noticed the kids counting the seats different ways and this was kind of a big aha for me when I realized that some of them weren't seeing the pattern really, they were just adding two and some of them were really seeing it differently when I was actually watching them and the way they counted them. So Jesse was counting one, two, three around like this and when I have done the problem myself that's always how I counted them. So it didn't occur to me there was another way to look at it and then Elisa was counting one, two, three, four, five like this and I thought she had twenty triangles out, so she's counted essentially you know, there is ten, you could then, then it looked totally different to me. I could see oh there is ten triangles on the top, ten triangles on the bottom, then there is a seat on either end. So now I was seeing the pattern a different way.

LB: I am just wondering now after the comments I am thinking why did we do one, two, three, four, you know, why did we do this pattern like that. I mean I know for myself it was…

Potential Learning by Teachers:

Provision of table influences what students learn from problem

Worksheet does not reveal what students know

Students may fill out worksheet correctly without understanding its relationship to problem

Students' counting can reveal their mathematical thinking

Use of textbook problems without critical consideration can be a problem

It's fun to see how

?: It was in there. LB: it was in there, right. So that's a good lesson for us I think always to really question just because it's already done for you, is it really the most effective use and I'm thinking how would the lesson have been changed if we had started off with you know ten and then seventeen and then you know just these random numbers, then you would not have had, you would not have, there would have been no vertical pattern. It's kind of fun to think about all the different things you can kind of tweak and then look at watch and see what you know, what they do. Gee, I guess that's called lesson study. JH: I think so.	changes in lesson affect student learning
SECOND TEACHING AND COLLOQUIUM **Title: Over the next two days, the group redesigns the lesson** **The worksheet is removed** **Each student has a unique number of tables** **Students share and write about findings** **Title: Second Teaching of Lesson: August 14, 2002** **Guest Teacher, Grade 4** S3: How do you get that S4 : six seats S3: no there's five seats S4: there's five tables S5: that's five tables S4: Okay that's five tables. S5: One, two S4: Look if I put it like this, S5: Long ways S5: one two, three, four, five, six, seven. S4: seven	The purpose of the lesson has changed from seeing the numerical pattern to understanding its relationship to the problem, i.e., being able to explain it in words

S3: See our pattern is right. S4: This is the pattern, there's only, how ever many tables there are, there are always two more seats S6: Every table gets one seat except for the top and bottom. S6: Every table gets one seat, S6: See, one, two, three, four, five, six, seven, eight, nine, ten, eleven. S7: Yeah S6: Number two. Who wants to write it? S6: (*writing on chart paper)* ….except the sides	
Title: All 6 group posters note the "plus two" pattern; data on individual students are not available **Title: Lesson Colloquium , Second Teaching, August 14, 2002** HC: I think this lesson probably got to more of the core of what we wanted to do which was to make the question more open-ended and really get the kids to understand the pattern, understand the rule as opposed to being able to plug in a number and get an answer in which we spoon-fed it to them, you know, with charts and the whole thing and it was kind of hard for us initially, you know we wanted that worksheet. And for us to get rid of it, I think that was really liberating for us just to think of okay how we are going to approach this to where we can really make sure they, they know the rule, they understand it and they know how they got it.	Positive description is interesting, given that she was very reluctant to remove worksheet. She's a 2nd year teacher and the instructor of the lesson.

Reflection after 2nd Teaching: What did we learn from revising and reteaching the lesson?	Self-monitoring: structure for recording changes in lesson, discussing was learned from revising, reteaching
August 14,2002 LB: Focus on the counting. Having the kids talk about their counting, that was a big improvement because we started to focus on the process of what's happening here. ES: Oh that's right. LB: That was an important improvement. JH: So having the students have to describe their counting really got at their thinking a lot more and also made the lesson more accessible to other kids. It gave other kids a lot of opportunities to hear you know and think about what was going on in the lesson. ES: That's exactly right. LB: And like just a personal aha for me when you had said that the counting, I don't know if it was you Jackie, somebody had said in the first debriefing that the counting, we should really, spend sometime on having them share that, you know, I thought, first, at first I thought, what, who cares about that. Did not see that as an important thing because I personally did not see the pattern that the 2 ends are the plus two. I never saw that. So it just shows that in all this math, well it's in everything we teach, that we only kind of as effective as the, our level of understanding. So we have always to keep pushing ourselves to delve into especially like in elementary grades, the stuff is really relatively simple. Like these kids today, plus three, plus three, but like, the why and how come, that's the challenge. JH: So if you had the answer to the sentence from teaching the lesson twice, we learnt that students…… DG: Need to do the work, not the teacher. LB: Yeah. ES: That's totally it.	Careful recording of student counting methods during first teaching provides basis for later learning LB allowed student sharing of counting methods to be added to lesson, although she didn't understand why this would be useful; what does this say about trust, group ownership? Strengthen commitment to own mathematics learning Students learn something from organizing the data themselves

Appendix A – Audience Response Form

Example Description: ___________________________

How important is the knowledge pursued by the teachers in this example?

How did features of lesson study support teachers' learning?

Did this example spark any ideas about whether the knowledge will be generative --- that is, whether it will lead to continued learning and improvement of practice over time? What do you think predicts fleeting vs. lasting learning?