Guide for the Videotape "To Open a Cube"

Synopsis of Videotape

Akihiko Takahashi teaches a public research lesson to fifth-grade students in the San Mateo – Foster City School District (CA), observed by 100 U.S. educators. The mathematics lesson is designed to build students' problem-solving skills and their understanding of three-dimensional geometry. Students are asked to solve a challenging, open-ended problem by making reasoned conjectures from their own prior knowledge, their investigations, and classmates' findings. The problem is: How many edges of a cube must be cut to make a "net" (a flat connected shape)?

Before Watching the Tape

Make a cube, cut it along the edges to open, and record how many cuts it takes to make the cube into a "net" (a flat, connected shape). There are several different "nets" (flat, connected shapes) that can be made from the cube. Does it take the same number of cuts to make each one? Why this might be so? What do you know about a cube, and how can that information help you figure out the number of cuts needed to open it into a net?

Discussion Points

Lesson Goals

- How might the lesson build students' understanding of three-dimensional geometry?
- How might the lesson build problem-solving skills and motivation?
- In what ways is the approach to mathematics learning represented in this lesson similar to or different from approaches familiar to you (e.g., approaches at your school, NCTM framework, etc.)?

Pedagogy

- What groundwork was laid to help students learn from one another and from their mistakes?
- How was students' prior knowledge elicited?
- How was a discussion among students facilitated?
- How were mistakes handled?

Data Collection

- What evidence did you see that students found this problem motivating?
- What evidence did you see that students built their capacity to solve problems?

General

- What questions would you like to ask the instructor?
- What aspects of the instruction, if any, would you like to incorporate in your own practice?

Detailed Description of Video

During the first part of the lesson, students discus the properties of a cube. This information is recorded on the board and misunderstanding corrected. Next students are asked how many edges must be cut in order to make the cube into a flat, connected shape ("net"). Students cut cubes, record findings, and post their data and cut cubes on the board. Students post two different answers (6 and 7 cuts); by re-counting, they discover that the nets labeled "6" have been miscounted. The lesson ends with the promise that students will investigate the problem further in the next mathematics class.

Following the public research lesson, the instructor and a panel of invited commentators conduct a lesson colloquium. An audience question period follows.

The videotape includes additional footage following the final credits. In this informal small-group discussion, Mr. Takahashi explains how he would have concluded the lesson had he not run out of time. He explains that he planned to conclude the lesson in the following way:

(1) ask students to consider whether the answer is always 7;
(2) ask students to notice what other features the nets all share (e.g.,they consist of 6 squares; 5 edges join the 6 squares);
(3) ask if these numbers will always be 6 and 5 (the cube has 6 faces; to be attached and flat, they must be joined by 5 edges); and
(4) ask students to think about connections between the number of edges that remain uncut (5), the number cut (7), and the total number of edges in a cube (12), i.e., that 12-5=7.

Panel Discussion

Background: A large public research lesson differs from a research lesson that takes place within an individual school (see "To lift 100 kilograms, for comparison). Commentators may focus on "big picture" issues relevant to the diverse schools where the attending teachers work. Panelists raise several issues:

- How lesson flow and student motivation are related
- Why the word "least" was included in the question "What is the least number of edges that need to be cut?"
- How students come to feel ownership of a problem
- That it is easy for a teacher to direct or lead students, but more powerful when students feel in charge
- That giving students the time to express their ideas and needs was essential to building student motivation, even though it kept the lesson from being concluded during this period

Additional Discussion Points from Panel

- Mr. Takahashi explains that he didn't want to rush students into proof before they were convinced that it had to be 7, and they really wanted to know why
- It is important for students to get "hungry" for deductive thinking, so they will understand how mathematics works

Audience Discussion

- Why were students asked to work in pairs? (To aid in careful counting)
- Discomfort was intentionally created in "6-7" discussion, so that students would feel the need for a definite solution
- Almost every student participated
- Blackboard used as careful record of lesson (e.g., the incorrect answer "24 corners" was not written)
- Students were engaged by problem, not by theatrics
- Why didn't each child have a cube during the discussion of its properties? (A concrete object makes it very easy to do and easy to forget)

About the Instructor

Akihiko Takahashi taught elementary school in Japan for 19 years, where he was nationally active in mathematics lesson study and mentored 200 pre-service teachers during his career. In 2002, he received his Ph.D. from the University of Illinois at Urbana-Champaign; his dissertation research focused on internet use in mathematics education. He is currently an assistant professor of mathematics education at DePaul University in Chicago.

Tell Us What You Think

Please share your comments, questions, and suggestions about this videotape with the Lesson Study Group at Mills College (lessonresearch.net or clewis@mills.edu). What was useful in this videotape, and what was problematic? What other materials would you like?