Teachers' Manual: Grade 3 Fractions
(Unit 16, 3B, pp.57-64)

1.Goals of Unit

Students will try to use fractions to express the size of a part smaller than one measurement unit.

Interest [1]

Students will try to use fractions to express the size of a part smaller than a measurement unit

Thinking

1. Students can explain that fractions express how many units of an equally partitioned quantity.
2. Students can explain why, for fractions that represent the same unit, it is correct to perform addition and subtraction using the numerators.

Expression

1. Students can use fractions to express a fractional part smaller than a measurement unit.
2. Students can perform addition and subtraction of fractions with like denominators in simple cases.

Knowledge

1. Students understand the meaning and written expression of fractions.
2. Students understand how to perform addition and subtraction of fractions with like denominators in simple cases.

2. Major points of unit

1) Expressing the size of a part smaller than one measurement unit

In unit 13 ("Decimal Numbers"), students studied how to quantify an amount less than a measurement unit, using decimals, which share the base-ten structure. The current unit introduces fractions, for which the unit can be created freely (i.e., any fractional part can be used as the measuring unit, see "*gojo-hou*" activity later in this manual).

In unit 13, students understood fractional quantities by using units of volume and length to measure and by partitioning these measurement units into 10 equal parts in order to express a part smaller than the measurement unit. But in the current unit of instruction, students will understand "$\frac{1}{2}$", "$\frac{1}{3}$" by dividing real tape material into equal lengths.

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[1] Japanese textbooks specify four areas of assessment: (1) Interest, Motivation and Attitude; (2) Mathematical Thinking; (3) Expression and Manipulation; (4) Knowledge and Understanding. These areas will be denoted throughout this document as Interest, Thinking, Expression and Knowledge, respectively.

Teachers need to help students understand that "fractions" are the way to express "an amount less than a measurement unit," not just a way to divide an amount into equal parts. Since at third grade the textbook has not yet introduced fractions greater than 1, students are likely to think about fractions as something that shows the relationship between a whole and parts. So students may identify fractions just as an operation that divides an amount into equal parts. Therefore, make sure students grasp the purpose of studying fractions [as a way to express an amount less than a measurement unit].

2) The size of fractions

From page 60, students consider fractions as a type of number, using a number line. Students gradually understand fractions as numbers, by observing structures such $\frac{2}{5}m$ is 2 pieces of $\frac{1}{5}m$, or 3 pieces of $\frac{1}{5}m$ is $\frac{3}{5}m$, and by comparing their sizes. However, formal study of fractions as numbers, not as measured quantities, is in the fourth grade. At this juncture, we limit our consideration of fractions as numbers to those fractions with 10 as the denominator [by relating them to decimal numbers.]

Finally, students will learn simple addition and subtraction of fractions with like denominators. This is simply to help students understand that fractions are numbers. It is important that students avoid just mechanically adding or subtracting, and that they notice why like-denominator fractions can be added or subtracted, by thinking about "how many unit fractions."

3) Teaching and evaluation plan

Subunit	Per	Goal	Learning Activities	Main Evaluation Points
1. How to express fractional part (page 57-59, 3 period)[2]	1 - 2	Students will try to use fractions to express the size of the part left over from measuring with a unit.	a) Think about how to express the length of 1 part of a 1m tape divided into 3 equal parts b) Understand that if you take one part of a meter divided into 3 equal parts, it is one-third meter, and written as $\frac{1}{3}m$. c) Think about how to express the length of 2 parts of a 1m tape that has been divided into 3 equal parts. d) Understand that 2 out of 3 equal parts of 1m is	(Interest) Try to find a way to express in meters the length of 1 out of 3 equal parts of a meter. (Knowledge) Understand that when 1m is partitioned into 3 equal parts, each part is called one-third of 1m and is written $\frac{1}{3}m$. (Expression) Can point out that $\frac{2}{3}m$ is 2 parts of 1m divided into 3 equal parts

[2] One lesson period in the Japanese curriculum is considered to be 40 minutes long.

			called "two thirds of 1m", and it is written $\frac{2}{3}m$.	
	3	a) Understand that volumes smaller than 1L can be expressed with fractions, as length was. b) Understand the meaning of "fraction" "denominator" "numerator".	a) Think about how to express 2 out of 5 equal parts of 1L. b) Think about how to express 1out of 4 equal parts of 1L, 4 out of 6 equal parts of 1L. c) Understand the meaning of the terms "fraction" "denominator" and "numerator".	(Expression) Use fractions to express the amount resulting from equal division of 1L. (Knowledge) Understand the meaning of "fraction" "denominator" "numerator".
2 .The size of fraction (page 60-63, 4 periods)	1	Understand that fractions can be expressed on the number line, and deepen understanding of the structure of fractions and their relative sizes.	a) Think how long are 2, 3, and 4 pieces of $\frac{1}{5}m$. b) Think which is longer, $\frac{4}{5}m$ or $\frac{2}{5}m$. c) Read $\frac{2}{6}L$, $\frac{4}{6}L$ indicated on the number line, and investigate 5 and 6 of $\frac{1}{6}L$.	(Expression) Use fractions to express quantities on the number line and recognize fractions marked on the number line.
	2	a) Using the number line, deepen understanding of the structure of fractions and their relative size. b) Understand fractions that do not have a measurement unit	a) Using the number line, understand the structure of fractions based on unit fractions. b) Looking at the fractions on the number line (tenths) think about the structure of fractions, their relative size, and the same-size decimals. c) Learn that the first decimal place is also called the $\frac{1}{10}$'s place.	(Knowledge) Understand the structure and relative size of the fractions of the unit $\frac{1}{10}$ shown on the number line, up to 1

	3	Understand and perform simple addition and subtraction of fractions (for which sum is smaller than 1, and the corresponding subtraction).	a) For the problem of finding the amount of $\frac{3}{5}L$ and $\frac{1}{5}L$ of juice combined, consider how to calculate $\frac{3}{5}+\frac{1}{5}$. b) Practice addition of fractions. c) For the problem of how much juice is left after taking away $\frac{1}{5}L$ from $\frac{4}{5}L$, think about how to calculate $\frac{4}{5}-\frac{1}{5}$. d) Practice subtraction of fractions.	(Thinking) Figure out that the fractions with like denominators can be added and subtracted in the same way as whole numbers, by thinking of fractions as how many of a unit fraction (Knowledge) Understand how to do simple addition and subtraction of fractions with the same denominator (the total amount of addition is less than 1, and the corresponding subtraction).
	4	Practice what students have studied in this unit.	a) Practice.	
Page 84 check (page 64, 1 period)		Confirm what they learned	Try "Check"	

Introduction of Fractions

Here, we introduce fractions using the easily grasped length of 1*m*, and we work on dividing 1*m* into equal lengths. However, if we ask students to divide 1*m* into separate parts at the outset, it is harder to emphasize the meaning of the left over part in relation to 1*m*. To help students understand the relationship between the part left over from measuring with a meter and the 1*m*, we set up an amount more than 1*m*. To draw out naturally the idea of equal partitioning, it is good to include an activity in which students predict the length of the left over part.

Subunit 1. How to express fractional part (page 57-59, 3 periods)

(The 1st and 2nd periods)

Goals: Students will try to use fractions to express the size of the part left over from measuring with a unit.

Preparation: Teachers: $1\frac{1}{3}m$ length of tape (one per student)

Students: Scissors

Suggestion: Teachers should put the one meter length point on the surface of the tape and mark the three equal parts of the m on the back because it might be difficult for some students to do the activities.

1. Look at the $1\frac{1}{3}$ *m* tape and predict the length of the "little more" over 1 meter, to become interested in the topic.

> ***(Key Question)***[3] *"This tape is 1m and a little more. How long is the "little more?"*

(Possible reactions) [4]

a. It is longer than 50cm. (half of 1m)

b. It is longer than 25cm. (quarter of 1m)

c. It is maybe about 30cm.

(Extra Support) When students respond "half" or "half of a half," use these responses to encourage students to think about dividing 1*m* tape into equal parts.

2. Have students grasp that the length of the "left over" is the same as the length of one part when 1*m* divided into 3 equal parts.

Cut the tape at 1*m*, and separate 1*m* tape and the "left over" tape. Then fold the 1*m* length of tape at the mark. (The length of every mark is $\frac{1}{3}m$. It should be the same length of the "left over.")

> ***(Key Question)*** *"Let's compare the "left over" with the length of 1m divided into 3 equal parts."*

(Possible reactions)

a. Students will unfold the 1*m* tape and compare the length of the left over part with the length of 1 out of 3 equal pieces of 1*m*.

b. Students will fold the 1*m* length of tape in thirds, and compare to the length of the left over part.

3. Help students grasp the meaning of today's lesson: to express in meter units the length of the piece left over from measuring with a meter.

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[3] These key questions, or "*hatsumon*" are the kinds of questions teachers may think of as key questions to pose to the students.

[4] The possible reactions shows examples of how students might respond to the key questions posed by teachers. The extra support suggests strategies to be used for students who are struggling with the problem.

(Interest)[5] Students try to find a way to express in meters the length of 1 out of 3 equal pieces of 1*m*. (Observation)

(Possible reactions)

a. One out of two equal pieces of 1m is called "one-half of a meter". Maybe 1 out of 3 equal pieces of 1m is called "one third of a meter."

(Extra Support) If students cannot come up with the idea of using fractions, teachers support the students through the thought process in "a" above.

4. Students learn that if you take one part of 1*m* divided into 3 equal parts, it is one-third *m*, and written as $\frac{1}{3}m$.

a. Refer to textbook page 58 line 1-4.

b. Instruct students how to read and write $\frac{1}{3}$.

(Knowledge) When 1m is partitioned into 3 equal parts, each part is called one-third of 1*m* and is written $\frac{1}{3}m$. (Notebooks)

5. Solve question 1) (parts (1) and (2)) on page 58.

> ***(Key Question)*** *"Let's write down not just the answer but also the reason why."*

(Expression) Students use fractions to express the length less than 1*m* shown by the tape. (Notebooks, Comment)

(Possible reactions to question 1) part (2))

a. If you divide 1*m* into 5 pieces, 1 of those pieces is called $\frac{1}{5}m$.

6. If 1*m* is divided into 3 equal parts, think about the length of 2 parts. (Individual solving)

> ***(Key Question)*** *When 1 meter is divided into 3 equal parts, what is the length in meters of two parts?"*

(Possible reactions) "1 piece of 1 meter is $\frac{1}{3}m$, so 2 pieces of it are $\frac{2}{3}m$"

7. Students understand that 2 out of 3 equal parts of 1*m* is called "two thirds of 1*m*", and it is written $\frac{2}{3}m$.

(Expression) Students can point out that $\frac{2}{3}m$ is 2 out of 3 equal pieces of 1*m*. (Notebooks, observation)

8. Solve question 1) at the top of page 59 and summarize this lesson.

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[5] Throughout this document, "Interest," "thinking," "expression, "and "knowledge" formatted in this way indicate assessment criteria teachers may want to be thinking of during their instruction.

Fractions and Partitioning

Among the students, some might understand "$\frac{1}{3}$ " just as partitioning of something into 3 equal pieces. If they can't go beyond this, it will be difficult to understand fractions as numbers, so you can have them use the tape to grasp the actual length of $\frac{1}{3}m$, and to understand the meaning of fraction as amount.

Introduction of $\frac{2}{3}$

Here, compare 2 of 3 equal pieces of 1*m* with the object of the same length, eliciting $\frac{2}{3}$ directly from the operation. The foundational idea that "it's 2 pieces of $\frac{1}{3}m$, so it's $\frac{2}{3}m$" is treated on Page 60 and following.

Introduction of fraction terms

Here, students learn the meaning of fractions using quantities such as 1"*m*"(meter) and 1 "*L*" (liter) where the quantity can be grasped clearly. These fractions express measured quantities. But when giving the definition of fraction, the measurement unit should be removed.

Supplementary problems

1) How many liters are there in 1 part if 1*L* is divided into 8 equal parts? How many liters if you had 5 of those parts or 7 of those parts? ($\frac{1}{8}L$, $\frac{5}{8}L$, $\frac{7}{8}L$)
2) If the numerator is 5 and the denominator is 6, what is the fraction? ($\frac{5}{6}$)

(3rd period)

(Goals of the 3rd lesson)

a. Understand that volumes less than 1*L* can also be expressed with fractions, as length was.
b. Understand the meaning of "fraction" "denominator" "numerator".

1. Grasp the meaning of problem 3, page 59.

Confirm students understand that the goal is to use a fraction to express an amount that is less than 1*L*.

2. Students will think about how to express 2 out of 5 equal parts of 1*L*. (Individual solving)

> ***(Key Question)*** *"How many liters of water are there? Let's explain why you thought so."*

(Expression) Use a fraction to express an amount resulting from equal partitioning of 1L. (Notebooks /Comment)

(Possible reactions)

a. Because 1*L* is divided into 5 equal parts, the first marker shows $\frac{1}{5}L$, the second marker shows $\frac{2}{5}L$.

b. Thinking as we did for meters, it's 2 pieces from dividing 1*L* into 5 equal pieces, so it's $\frac{2}{5}L$.

(*Extra Support)* For students who are stuck, help them think about the quantity shown by the first mark on the water container.

Help students confirm that 2 out of 5 equal pieces of 1*L* is $\frac{2}{5}L$.

3. Solve question 1) on page 59.

4. Learn the words "fraction" "denominator" "numerator" and understand their meaning.

"Numbers like $\frac{1}{3}m$ and $\frac{2}{5}L$ and $\frac{1}{3}$ and $\frac{2}{5}$ are called fractions. And they have denominators and numerators. The numbers under the line in every fraction is called the denominator, the number above the line is called the numerator."

5. Solve questions 2) and 3) page 59.

(Knowledge) Understand the meaning of "fraction", "denominator", "numerator". (Comment, Notebooks)

6. Summarize this lesson with students.

(Thinking) Notice fractions are a useful way to express the part left over from measuring with a unit.

How to grasp fractions through unit fractions.

Here, through comparison with whole number thinking such as "2 pieces of 1*m* is 2*m*" and "3 pieces of 1*m* is 3*m*", students grasp the structure of fractions using unit fractions, such as "2 pieces of $\frac{1}{5}m$ is $\frac{2}{5}m$", "3 pieces of $\frac{1}{5}m$ is $\frac{3}{5}m$".

Among students, some come up with incorrect answers such as "$\frac{4}{5}m$ is 3 pieces of $\frac{1}{5}m$", "5 pieces of $\frac{1}{10}$ is $\frac{6}{10}$". These problems happen because students

don't yet clearly understand unit fractions. So, to help students understand the multiplicative structure of unit fractions, teachers need to use the number line and tape diagram.

Possible responses of students and assessment

1) $\frac{4}{5}$ is 4 pieces of $\frac{1}{5}$, $\frac{2}{5}$ is 2 pieces of $\frac{1}{5}$, and $\frac{4}{5}$ places to the right of $\frac{2}{5}$ on the number line, so $\frac{4}{5}$ is larger than $\frac{2}{5}$. (Student understands fractions based on the meaning of unit fractions and number line.)

2) We know how many pieces of $\frac{1}{5}m$ by counting numerators. So we can judge that $\frac{4}{5}m$ is longer than $\frac{1}{5}m$. (Student understands the meaning of numerator)

Subunit 2. The size of fractions (page 60-63, 4 periods)

(1st period)

(Goals of 1st period)

Through understanding that fractions can be represented on a number line, deepen understanding of the structure of fractions and their relative size.

Preparation: paper

1. Think about the length in meters of 2, 3, and 4 pieces of $\frac{1}{5}m$.

> ***(Key Question)*** *"How long are 2 pieces of $\frac{1}{5}$m?" (Continue to question how long are 3, 4 pieces of $\frac{1}{5}m$)*

2. Illustrate $\frac{2}{5}m$, $\frac{3}{5}m$ and so forth on number line.

(Expression) Students express fractions on number line. (Paper)
(Extra Support) Help them think about the correspondence of the tape diagram to the number line.

> ***(Key Question)*** *"Where is 5 pieces of $\frac{1}{5}$m on the number line?"*

- 5 pieces of $\frac{1}{5}m$ are $\frac{5}{5}m$ and it become 1m (page60,1),(2))

3. Think which is longer, $\frac{4}{5}m$ or $\frac{2}{5}m$.

(Key Question) *"Which is longer, $\frac{4}{5}$m or $\frac{2}{5}$m? Let's think about the reason, too."*

(Thinking) Students can judge the relative size of a fraction by thinking about how many of a unit fraction are in it. (Observation and Comment)

4. Solve the questions page 60. 1).

(Expression) Students read and understand the fractions on the number line. (Notebooks)

5. Summarize this lesson

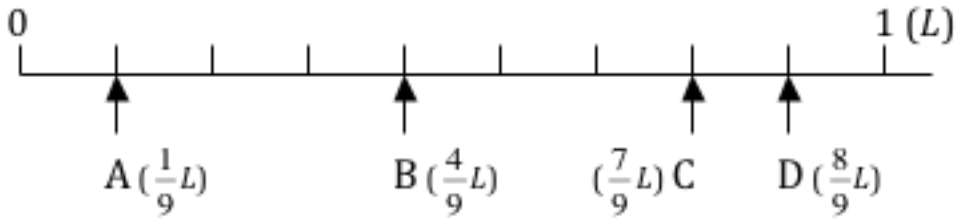
Summarize the idea that fractions can be expressed on the number line, like whole numbers and decimal numbers.

How to understand fractions that do not have measurement units (*m*/*L*)

Here, students first learn fractions as abstract numbers without measurement units. It is confirmed that "the first decimal place" studied in Unit 13 is the same as $\frac{1}{10}$ths. In this grade students learn the relationship between decimals and fractions, using only fractions with the denominator of 10. The main study of fractions as numbers will occur in fourth grade, so teachers should not go beyond the third grade learning objectives here.

Supplementary questions

1. On the number line below, how many liters are shown at each mark? ($\frac{1}{9}L$, $\frac{4}{9}L$, $\frac{7}{9}L$, $\frac{8}{9}L$)



2. Fraction size (2nd period)

Goals:

1. Using number line, deepen understanding of the structure and relative size of fractions.
2. Understand fractions that do not have measurement units associated with them (such as *m* or *L*).

1. Using the number line, understand fractions as numbers.

(Knowledge) Understand fractions made up of $\frac{1}{10}$ths up to 1. (Notebooks)

(Key Question) *"Do you notice anything on the number line?"*

(Possible reactions)

a. Those fractions do not have the units of *m* or *L.*

b. We have never seen fractions with the denominator 10.

- The fractions do not have measurement units, and are treated as numbers

(Key Question) *"How many $\frac{1}{10}$ do you need to make $\frac{3}{10}$ and $\frac{5}{10}$?"*

With number line, make sure that $\frac{3}{10}$ needs 3 pieces of $\frac{1}{10}$, $\frac{5}{10}$ needs 5 pieces of $\frac{1}{10}$. Based on this knowledge, have students figure out the answer of what number is 9 pieces of $\frac{1}{10}$. Moreover, have students understand that 10 pieces of $\frac{1}{10}$ become $\frac{10}{10}$, which means 1.

2. Express $\frac{1}{10}$'s place in decimal number.

3. Students learn that the first decimal place is also called the tenths place.

4. Solve problems 1), 2) on page 61

5. Summarize this lesson.

Addition and subtraction of fractions

The main purpose of addition and subtraction of fractions in this unit is to help students recognize fractions as numbers by seeing that they can be added and subtracted. Therefore, teachers do not have to finalize students' knowledge of how to add and subtract fractions with like denominators. (We will do this in fourth grade.) But if students understand that concept well, teachers can try. If we think of them as units, fractions and decimals can be added and subtracted in the same way as whole numbers; coming to this understanding provides a good opportunity for students to notice the merits of mathematical reasoning.

Possible responses

a. Figure out the answer by looking at the picture. (There are 3 pieces of $\frac{1}{5}$ *L* and 1 piece of $\frac{1}{5}L$. So we have 4 pieces of $\frac{1}{5}$. *L* So in all, it will be $\frac{4}{5}L$)

b. Notice the numerators. You know how many $\frac{1}{5}L$ from looking at the numerator.

So, 3+1=4, so $\frac{4}{5}L$)

c. Figure out the answer using the number line.

(3rd period)

Goals:

Understand and perform simple addition and subtraction of fractions (for sums less than 1, and their inverse for subtraction).

1. Read question 3, page 62 and make sure students know what is being asked in the question. (How much juice is there altogether?)

2. Think about how to write a math sentence for this problem. (Individual solving)

> ***(Key Question)*** *"What is the math sentence to solve this problem?"*
>
> ***(Key Question)*** *"Let's explain why the math sentence* $\frac{3}{5}+\frac{1}{5}$ *is correct"*

(Possible reactions)

a. Question is asking the total amount of water in containers altogether, so we use addition.

b. We can add $\frac{3}{5}$ L and $\frac{1}{5}$ L like we can add 1L and 2L.

3. Think about how to do the calculation.

(Thinking) Students figure out that if they think of fractions as "how many pieces of a unit fraction," they can calculate like-denominator fractions in the same way as whole numbers. (Comments)

(Extra Support) Have students notice how many $\frac{1}{5}L$ there are in $\frac{3}{5}L$ and how many in $\frac{1}{5}L$.

4. Present and discuss their thinking.

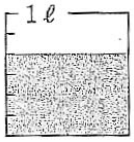
(Knowledge) Understand the addition of fractions whose denominators are the same. (Notebooks/ Discussion)

5. Explain how to calculate $\frac{2}{5}+\frac{3}{5}=1$

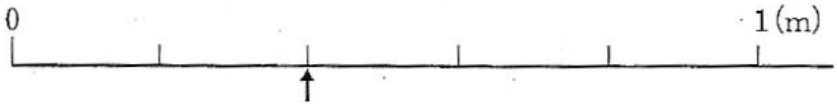
6. Work on calculation problems (1), p.62.

7. Read question 5, page 62, and make sure students understand what is being asked in the question. (How much juice will be left when $\frac{1}{5}L$ **is taken away from** $\frac{4}{5}L$**?)**

Supplementary questions

1. In the picture at the right, how much is in the 1*L* 

container?
2. Let's think using the number line below.



1) How many meters does the arrow show?

2) How much more do you need to add to $\frac{2}{5}m$ to reach 1m?

3) Write the appropriate symbol in the blank. (Equal, greater than, less than)

$\frac{2}{4} \square \frac{3}{4}$ $1 \square \frac{10}{10}$ $\frac{4}{5} \square \frac{3}{5}$

8. Think about how to calculate $\frac{4}{5} - \frac{1}{5}$

(Thinking) Students figure out that the fractions with the same denominator can be calculated like whole numbers, if they think of fractions as how many pieces of a unit fraction. (Discussion)

(Extra Support) Help students notice how many pieces of $\frac{1}{5}$ are in $\frac{4}{5}$ and $\frac{1}{5}$.

9. Present and discuss thinking.

10. Explain the calculation $1 - \frac{3}{5} = \frac{2}{5}$

11. Solve page 63, 6 1)

(4th period)

Goals:

Apply and practice learning from this unit.

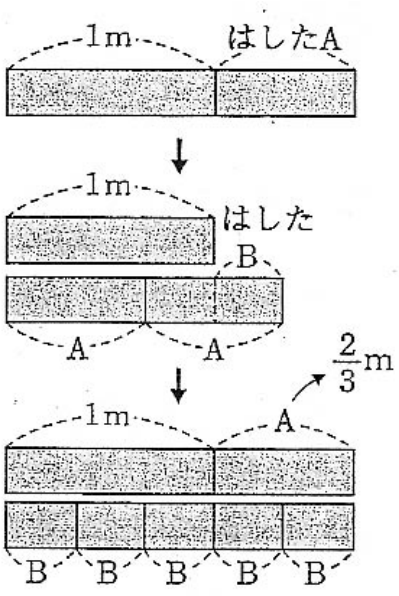
1. Problems on how to express volumes less than 1*L* in fractions.
2 .Express designated fractional lengths on a tape diagram.
3 .Problems of addition and subtraction of same-denominator fractions.

Another activity "gojo-hou"

Fractions are numbers that originated from the operation "*gojo-hou.*"" *Gojo-hou*" is repeatedly measuring using a fractional part left over from measuring until there is no fractional part left over. In third grade, we could have started the study of fractions using this method, but unit fractions like $\frac{1}{3}$ are measured with only one operation of "*gojo-hou,*" (i.e., partitioning the meter into three equal $\frac{1}{3}$ meter pieces) and we do not have to repeat it (i.e., measure again with smaller fractional

parts) after that. So students will not understand the usefulness of "*gojo-hou*" at that point. It is more effective to explore "*gojo-hou*" at the end of this unit, when it will deepen understanding of the meaning of fractions.

As shown in the picture at the right, *gojo-hou* is using the leftover part to measure the prior fractional part, and doing this repeatedly until there is no leftover.



Check (page 64, 1 period)

1st period

Goals: Confirm understanding of what has been learned in the unit

1. Problems that assess students' ability to express volume of less than 1*L* using a fraction
2. Problems that assess students' ability to represent fractions on number line.

Challenge (page 64, no period allocated)

Goals: Students will develop their interest in mathematics and an attitude of initiative as learners by making use of their knowledge that they have learned in this unit.

Let's make Fraction Rulers!

We can make fraction rulers by following the illustration in the textbook. If you want to divide 1*m* into 7 equal pieces, put the edge of your tape on the first line end, eighth line of the notebook, and put the marks on every crossed line. In third grade, the length of tape which is used in the problem is fixed, to prevent student confusion. But with this idea here, students will learn how to divide line into any number of equal partitions they want.

Research Volume of Teachers' Manual
Explanation of Unit 16 and Points for Instruction

The beginnings of studying fractions

One part of a quantity divided into 3 equal parts is called "one third" of the whole, and is written $\frac{1}{3}$. When we use $\frac{1}{3}$ in this way to talk about $\frac{1}{3}$ of a real object divided into fractional parts, $\frac{1}{3}$ is called a part-whole fraction.

A part-whole fraction naturally leads to multiplication and division, since $\frac{1}{3}$ of 27 can be solved as 27÷3. However, a part-whole fraction does not qualify as a number because size comparison is not meaningful and addition and subtraction cannot be performed.

For that reason, it is often considered best to avoid dealing with part-whole fractions if possible during the introduction of fractions, so that students will develop a concept of fractions as numbers.

However, in this introduction, fractions are used to express an amount smaller than one unit, using equal partitioning of the unit. Considering this and also the future study of fractions, completely avoiding part-whole fractions may not be possible. If so, it is best not to go deeply into part-whole fractions, and to shift as quickly as possible to teaching fractions as numbers.

Challenges of studying fractions

If students haven't understood fractions well, it is often not discovered until later grades. Examples 1 and 2 are from upper grade students. Both answers should be "$\frac{1}{3}$". But some students answer $\frac{1}{4}$ (for number line 1) or $\frac{1}{6}$ (for number line 2). It appears that they do not understand that 1*m* is considered a base amount to divide into equal parts. They misunderstand what to divide, and divide the number line illustrated as a whole.

This misunderstanding is probably caused by their initial instruction in fractions. When teachers introduce fractions, they often have students demonstrate the operation of dividing an amount into some number of equal parts. In this way, fractions are treated as part-whole fractions and the base amount is the whole. So students who are strongly impressed by this concept may identify the whole number line as the base amount instead of 1*m* as the base amount, as in examples 1 and 2.

To avoid this problem, teachers need to help students understand that the base amount is not always the whole. The use of the standard units 1*m* or 1*L* as the base amount and the use of number lines to think about fractions are methods to help students develop an understanding of fractions as numbers as soon as possible.

Teaching addition and subtraction of like-denominator fractions

In the third grade students learn that fractions can be added and subtracted and that ideas from addition and subtraction of whole numbers and decimals can be applied to addition and subtraction of fractions. Through activities such as addition of subtraction of fractions, students better understand fractions, especially the composition of fractions. For instance,

when $1L$ is divided into 5 equal parts, one part is $\frac{1}{5}L$. $\frac{3}{5}L$ is 3 parts of $\frac{1}{5}L$ (triple), $\frac{4}{5}$ is 4 parts of $\frac{1}{5}L$(4 times). So we can make equations such as $\frac{3}{5}L+\frac{1}{5}L$, $\frac{4}{5}L-\frac{1}{5}L$. These are the exactly the same way we add and subtract with whole numbers.

When practicing calculation of fractions, students learn about fractions, especially the composition of fractions. In third grade, students learn addition and subtraction of amounts smaller than 1, using mainly illustrations and the number line. In fourth grade, students will learn more difficult calculations including amounts larger than 1. Materials such as tape or water that help students understand should be used, especially in third grade.

Teachers should guide students carefully as noted, and try to avoid procedural practice of fraction calculation.

1.Goal

Students will try to use fractions to express the size of the fractional part left over from measuring with a unit.

2. Assessment Criteria

(Interest) Students think about how to express in meters of the length of 1 part of a meter partitioned into 3 equal parts.

(Thinking) When 1 meter is partitioned into 3 equal parts, each part is called one third of 1 meter. By using this knowledge, think about how long 2 pieces of a meter divided into 3 equal pieces would be.

(Knowledge) Know that one piece of a meter partitioned into 3 equal pieces is called “one third” of a meter, and know how to write it as $\frac{1}{3}$ *m*.

3.Teaching Points

Introduce fractions that have certain units such as meter or liter.

This is the first time to introduce the concept of fractions. “Fraction” has several meanings but there are two things indicated to teach in the teachers’ framework. One is “to understand the fractional part left over from measuring with a unit”, the other is “to understand the amount divided into equal parts”. In unit 16, teachers have to teach both. If students only had to understand fractions as equal partitioning of an amount, then an apple or piece of paper would provide sufficient

teaching material. But apples and paper are not sufficient to help students understand the fractional part left over from a measurement unit.

Using a tape that is longer than 1 meter, students try to express in meters the fractional part left over from measuring with a meter, and they understand that the fractional part is the length of 1 part when 1*m* is partitioned into 3 equal parts. Students thus learn both these concepts of fractions.

4. Lesson Plan

Lesson and Key Question	Learning Activities and Reactions	Note and Main Evaluation Point.
1. Create interest in the topic. "Let's compare the "little bit extra" with the length of 1*m* divided into 3 equal parts."	Students will look at the tape and consider the length of the fractional part (Possible reactions) 1 "It is longer than 50*cm*.(half of 1*m*) 2"It is longer than 25*cm*.(quarter of 1*m*) 3 "It is maybe about 30*cm*".	Teachers have students close their textbooks and show $1\frac{1}{3}$ *m* tape. Then teachers separate the tape into 1*m* and remainder part. (Technique) When students respond "half" or "half of a half," use these responses to encourage thinking about proportions of the 1*m* tape.
2. Understand the length of the fractional part. "Let's fold the 1*m* length of tape into three." "Let's fold the 1*m* length of tape into 3, and compare to the length of the fractional part.	Students understand that the length of the fractional part here is 1*m* divided into 3 equal parts.	Give every student a 1*m* length of tape and a $\frac{1}{3}m$ length of tape. Then have them fold the 1*m* length of tape into three. Since it might be difficult for students to handle it by themselves, teachers should mark 3 equal parts.
3. Grasp the meaning of the topic "Let's think about how to express the fractional part in meters.	Grasp the meaning of the topic so that they can express the fractional part in meters.	Place emphasis on "dividing into 3 equal lengths."
4. Students will try to find the way to express the length of one out of three equal pieces of 1*m* in meters.	Students will try to find the way to express the length of one out of three equal pieces of 1*m* in meters.	Students will try to find the way to express the length of one out of three equal pieces of 1*m* in meters.

1. Goal

1) Understand that amounts smaller than 1*L* can be expressed as fractions.
2) Understand the meaning of “fraction”, ”denominator”, ”numerator”.

2. Assessment Criteria

(Interest) Based on previous learned knowledge (length by using tape), express the amount that is smaller than 1*L*.
(Thinking) Expect to notice that “fraction” is the useful way to express the amount that is smaller than 1*L*.
(Knowledge) Understand the meaning of “fraction”, ”denominator”, “numerator”

3. Teaching point

Confirming the Meaning of Fractions as Equal Parts

After the previous learning of fractions using tape, here, students will learn the concept of fractions using volume-material such as water. Teachers need to help students see the shared concept that underlies measuring the marked volume of water and the length of tape.

Definition of Fraction

In this lesson, students learn for the first time the meaning of “fraction”, ”denominator”, and ”numerator”. Students should not simply be given these terms procedurally; they need to understand that the denominator is the number of equal partitions of the unit and the numerator is how many of these are being expressed. Here, the definition of fraction occurs without a measurement unit.

4. Lesson Plan

Lesson and Key Question	Learning Activities and Reactions	Note and Main Evaluation Point.
1. Understand this topic. "How to express an amount of water, which is less than 1L?"	Figure out the question on page 59.3 Teachers confirm students understand that this lesson's goal is to express an amount which is less than 1L, using a fraction. Based on the previous lesson, students will imagine that they can express an amount of water using a fraction. Beyond length in previous lesson, 1L can be divided into some equal parts and we can investigate how many portions of the amount there are. Students will think about how to express 2 of 5 equal parts of 1L.	Understand that a volume smaller than 1L can also be expressed with a fraction, as length was.
2.Understand that a volume smaller than one liter can be expressed with a fraction, as length was. "Express by fractions and explain why you thought like that"	1-Because 1L is divided into 5 equal parts, the first marker shows $\frac{1}{5}$ liter, the second marker shows $\frac{2}{5}$ liter. 2-Elicit the answer here with the same method as previous sections. Two out of 5 equal pieces of 1meter was two-fifths meter. So two out of 5 equal pieces of 1liter is two-fifths liter. Have discussion about students' opinions. (reactions) refer to above.	(Interest) Understand that a volume smaller than one liter can also be expressed with fractions, as length was. Teachers help them think constructively about how many equal parts 1L is divided into, and how much water is shown by each interval? Summarize that 2 out of 5 equal parts of 1L is $\frac{2}{5}L$. It is found out by the same way previous lesson.
3. Have discussion about students' opinions. "Express by fractions and explain why you thought like that"		

1.Goal

Understand that fractions can be expressed on the number line, and recognize the composition of fractions [i.e., that non-unit fractions are composed of unit fractions] and their relative size.

2.Criteria of evaluation

(Interest) Students are expected to think about how to express fractions on a number line.

(Thinking) By thinking "How many unit fractions are in that fraction?" learn to judge the relative size of fractions.

(Expression) Express fractions on the number line and grasp the size of fractions on the number line.

(Knowledge) Understand the composition and relative size of fractions.

3.Teaching point

Here, students have to learn the way of identifying fractions as how many unit fractions are in it. This knowledge is important not only to express fractions on a number line, but also to recognize the size of fractions. For instance, students are expected to grasp that "$\frac{4}{5}m$ is longer that $\frac{2}{5}m$. Because $\frac{4}{5}m$ is 4 pieces of $\frac{1}{5}$ and $\frac{2}{5}$ meter is 2 pieces of $\frac{1}{5}$. Which is a larger amount of $\frac{1}{5}$ pieces?"

Moreover, depending on the class, the teacher can explain to students that numerator expresses how many pieces of a unit fraction here and you can judge which fraction is larger or smaller (when denominator is the same number) by the value of the numerator.

4.Lesson Plan

Lesson and Key Question	Learning Activities and Reactions	Note and Main Evaluation Point.
1. We can judge the size of a fraction by how many of a unit fraction it has. " How long are 2 pieces of $\frac{1}{5}m$?" (continue to question how long are 3,4 pieces of $\frac{1}{5}m$) 2. Understand this topic. "Let's mark $\frac{2}{5}m$ or $\frac{3}{5}m$ on the number line.	Think about how long are 2 pieces of $\frac{1}{5}m$? 2 pieces of $\frac{1}{5}m$ are 2 pieces of $1m$ divided into 5 equal parts. So we can say $\frac{2}{5}m$. Recognize that the main goal of this topic is express $\frac{2}{5}m$, $\frac{3}{5}m$, $\frac{4}{5}m$ on the number line.	Teachers have students close textbooks and write the questions on the blackboard or paper. Teachers make sure that each answer of the question is $\frac{2}{5}m, \frac{3}{5}m, \frac{4}{5}m$ Give students papers which have illustration of page 60,1)

1.Goal

Understand and practice simple addition and subtraction calculations with fractions where the total amount of addition or subtraction calculation is smaller than 1.

2.Criteria of evaluation

(Interest) Try addition and subtraction of fractions.
(Thinking) How many pieces of a unit fraction are in the fraction? Based on this knowledge, students are expected to notice that addition and subtraction with fractions sharing a denominator is done in the same way calculations are done with whole numbers.
(Express) Perform addition and subtraction of fractions that have the same denominator.
(Knowledge) Understand and practice simple addition and subtraction calculations with fractions where the total amount of addition or subtraction calculation is smaller than 1.

3. Teaching point

In unit 16, the ultimate goal of learning calculation here is to grasp that fractions are numbers. So using the fractions that previously appeared in the textbook, students will try to think about how to calculate fractions. And students will learn that, as for whole numbers, calculations can be performed with fractions. Among the students, some will notice that fraction calculation can be done by adding and subtracting the numerator. But this should be taught according to the level that the students understand.

4.Lesson Plan

Lesson and Key Question	Learning Activities and Reactions	Note and Main Evaluation Point
1.Understand this topic. "Do you notice anything on the number line?"	Organize the information in the question and make it clear that what the question is that students need to answer. a) Information in this question. "There is $\frac{3}{5}L$ of juice in a carton and $\frac{1}{5}L$ in a bottle. b) What is the question? "How much juice is there altogether?"	Teachers should write questions and illustration on the blackboard or paper.
2.What is the equation like? "Let's make equation to	Referring to the illustration, students try to figure out	Teachers try to have students remember that they

answer the question of how much juice is there altogether?	the correct equation in order to answer the question. "Here, we can answer the question by addition. So $\frac{3}{5}+\frac{1}{5}$.	used square graduated container in second grade. They learned addition there, so they can apply the knowledge here, too. Teacher should use real square graduated container for lesson here, if possible.
3. Have discussion. "Explain that why you can use addition in this question?"	"Let's explain why equation $\frac{3}{5}+\frac{1}{5}$ is correct?" 1) Question is asking the total amount of water in containers altogether, so we use addition. 2)We can add $\frac{3}{5}$ *L* and $\frac{1}{5}$ *L* like we can add 1*L* and 2*L*.	Students are expected to figure out that the fractions whose denominators are the same number can be calculated in the same way as whole numbers, if they understand fractions as how many pieces of "*tani-bunsuu*" exist?

3. Key points of the activity

Summary of the activity

If you try to make a ruler that measures in $\frac{1}{7}$ meters, it will be hard to do this by folding (as you could for $\frac{1}{4}$ meters), because you can't fold exactly in the length of $\frac{1}{7}m$. The picture in the textbook explains how to make a ruler that measures in $\frac{1}{7}$ meters but some students may have trouble understanding it. So prepare a lined paper of 1*m* x 1*m* and move a tape strip on the paper [so that it includes seven intervals]. Students can measure the things around them with the rulers they make. As they measure various things, students will recognize that the need for rulers that measure other lengths. So the teacher can guide them to make rulers that measure other lengths.

Students can enjoy various activities using $\frac{1}{7}m$ rulers such as making presents for new first grade students, or making a calendar where they put pictures and dates of school events.

What is the benefit of a fraction ruler?

To measure the fractional part left over from measuring with a unit, we need various rulers according to the lengths we want to measure. Here, after learning how to make $\frac{1}{7}m$ length ruler, students need to learn the benefit of making rulers to measure various lengths. In these activities, some students might have questions about why [the strategy of using lined paper] enables them to partition into equal parts. It is difficult to explain to third-graders because they haven't learned the concept of "parallel" and "expansion and reduction". Teachers need to help students make sure that the rulers they have made are properly divided into equal parts.